I have been teaching separation of variables for over 30 years, and I have always been instructing students
to look for a self-adjoint problem first. We know then that the separation constant (the eigenvalue) is real
and, if the operator happens to be positive definite, we also know that the constant is positive. This a-priori knowledge about the separation constant simplifies greatly the solution of the problem.
More importantly, by the Sturm-Liouville theory (Spectral Theorem for Self-Adjoint Operators), we know
that the eigenvectors form simultaneously a basis for the L2 as well H1 space, which opens up the way
for a rigorous well-posedness proof [2].

Only recently, when studying acoustical and electromagnetic waveguides, we have run into a situation where
one needs to perform the separation of variables with a non-self adjoint operator [3,4]. This has led me to
a half-year long study of excellent and fundamental book by Gohberg and Krein [1] and a number of fundamental
results that I have learned (and should have known a long time ago).

In the talk, I will illustrate the deep theory for non-self adjoint operators with a simple model acoustic
waveguide problem with impedance boundary conditions.

[1] I. Gohberg and M. Krein, Introduction to the Theory of Linear Nonselfadjoint Operators in Hilbert
Space (Translations of Mathematical Monographs). American Mathemtical Society, 1965 (Russian edition), vol. 18.

[2] M. Melenk, L. Demkowicz, and S. Henneking, "Stability analysis for electromagnetic waveguides. Part 1: Acoustic and homogeneous electromagnetic waveguides.," Oden Institute, The University of Texas at Austin, Austin, TX 78712, Tech. Rep. 2, 2023, in review.

[3] L. Demkowicz, M. Melenk, J. Badger, and S. Henneking, "Stability analysis for acoustic and
electromagnetic waveguides. Part 2: Non-homogeneous waveguides.," Advances in Computational Mathematics, 2024, accepted, see also Oden Institute Report 2023/3.

[4] L. Demkowicz, N. Heuer and J. Gopalakrishnan, ``Stability Analysis for Acoustic Waveguides.
Part 3: Impedance Boundary Conditions", Oden Institute Technical Report 2024/1, submitted to
Applications of Mathematics.

Bio:

Leszek F. Demkowicz is Assistant Director of the Oden Institute for Computational Engineering and Sciences and holder of W. A. ``Tex" Moncrief, Jr. Chair in Computational Engineering and Sciences II. He is a Professor in the Dept. of Aerospace Engineering and Engineering Mechanics and a Professor in the Dept. of Mathematics, at The University of Texas at Austin. He has a M.S. in mathematics from Jagiellonian University, and M.S., Ph.D. and Sc.D. degrees in mechanics from Cracow University of Technology (CUT), Poland.

Dr. Demkowicz authored a monograph on adaptive methods (in Polish, 1986), co-authored with J.T. Oden a textbook on Functional Analysis (CRS Press, 1996, 2010, 2017) , a two-volume monograph on ``Computing with hp-Adaptive Finite Elements" (Chapman \& Hall/CRC, 2006,2007), Lecture Notes on Energy Spaces (Austin 2018) and Lecture Notes on the Mathematical Theory of Finite Elements (Austin 2020). He has authored and co-authored over 220 journal articles an other publications in the general area of computational mathematics and mechanics.

His work and scientific interests span across numerical analysis, adaptive finite element methods, wave propagation problems, including acoustics, elastodynamics and electromagnetics, and CFD. In the last decade his work focused mainly on the Discontinuous Petrov Galerkin Method, co-invented with Jay Gopalakrishnan from Portland State University. His recent research has been sponsored by NSF, Air Force, Navy, Army, DOE and Sandia Labs.

For his research on higher order methods, he has been awarded Zienkiewicz Medal by PACM, Computational Science Award by USACM, ICES Distinguished Research Award, and the Computational Mechanics Award by IACM in 2014. He is a Fellow of both IACM and USACM
and an honorary member of PACM. Since 2014, he is a foreign member of Polish Academy of Arts and Sciences.

He has graduated 25 Ph.D. and 7 M.S. students, and is currently supervising 2 Ph.D. students.