Basis of a stable conforming approximation by the DPG methodology

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The Petrov-Galerkin method is a generalization of the Galerkin idea (1914, also known as the Bubnov-Galerkin method), in which one utilizes different trial and test spaces. The Petrov-Galerkin method was proposed by Petrov in 1959 in the context of a 1D boundary-value problem. We will focus in the talk on the conforming approximation, i.e. such that the finite dimensional spaces are subspaces of the infinite dimensional counterparts ($U_h \subset U,\ V_h \subset V$).

Several years ago Bottasso, Causin, Micheletti and Sacco introduced the concept of the Discontinuous Petrov Galerkin (DPG) method. This method uses the ultra-weak variational formulation, which involves a non-symmetric functional setting with both discontinuous trial and test functions from different spaces. Both sets of functions are predefined a-priori. They struggled with stability and were forced to use stabilization terms. This method is also called the hybridized approach (HDG), since it allows certain inter-element numerical traces to be unknown.

A few years ago Demkowicz and Gopalakrishnan proposed a new confirming DPG finite element methodology with a built-in stability due to an on the fly computation of optimal (in practice, almost optimal) test functions that provide stability without extra stabilizing parameters, even in the presence of sharp gradients or discontinuities and for arbitrary length scales. It may be applied to arbitrary variational formulation. However, the versions with broken test spaces are recommended for practical applications since in such cases the test functions may be computed element-wise. Therefore, the word discontinuity refers only to the test functions. This method has been rapidly developed in recent years, since besides the unconditional stability it offers an a posteriori error estimate and always produces symmetric positive definite stiffness matrices.