DPGstar Method

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Following the philosophy of Cohen, Dahmen and Welpert in [3], we replace at the continuous level a variational problem with the mixed problem involving an additional unknown: Riesz representation of the (zero) residual corresponding to the original equation, equal to zero as well. The original problem may be replaced with its counterpart using the broken test space [2]. The entire existing DPG technology can then be interpreted as a stable discretization of the mixed problem.

If we place the load in the second equation in the mixed formulation, we arrive at the dual problem. The new unknown is no longer zero but it represents the solution of the dual problem. We call the discrete version of the dual problem the DPGstar method.

The theory for DPG discretizations automatically covers the theory for the DPGstar method as well. If the DPG method teaches us how to solve an overdetermined system of equations, the DPGstar provides meaning to an underdetermined one. For the strong variational formulation, DPGstar coincides with the well known FOSLLstar method [1].

We will compare the two methods numerically and discuss the relevance of the two formulations in context of goal-oriented adaptivity, see [4] for details.

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[3] A. Cohen, W. Dahmen, and G. Welper. Adaptivity and variational stabilization for convection-diffusion equations. ESAIM Math. Model. Numer. Anal., 46(5):1247--1273, 2012.

[4] L. Demkowicz, B. Keith, and J. Gopalakrishnan. DPG* method. Technical Report 24, ICES, 2017.