This will be a loose story about several very fundamental tools whose understanding I had recently to revise when studying the so-called 'open waveguide poblems'. It all starts with proper understanding of perhaps the most common elementary technique for solving PDEs in separable domains - the separation of variables. We teach that when we separate the variables, we must obtain a Sturm-Liouville problem in one of them. This gives us an a-priori information about the separation constant, and we start the solution from there. But (classical) Sturm-Liouville applies only to a bounded interval and, hence, the technique applies only to bounded domains. The knowledge of the general Spectral Theorem for Unbounded Operators opens a new door - the resolution of identity may now include both a series of eigenvectors and an integral part - Fourier-like transform involving pseudo-eigenvectors and the continuous part of the spectrum. Indeed, the Fourier transform is just a special case and we have thousands of Fourier-like transforms.

Time permitting, I may continue with a couple of more involved subjects: computation if inverse transform by using Residue Theorem and a relation with so-called leaky modes, and analysis of bent waveguides.