20th Inter-Institute Seminar for Young Researchers

Jubilee Session for Professor Zenon Waszczyszyn



Programme and Book of Abstracts

> October 9-10, 2015 Cracow, Poland



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Editors:

A. Matuszak J. Pamin

Organizers:

Cracow University of Technology Faculty of Civil Engineering Institute for Computational Civil Engineering

Organizing committee:

J. Pamin A. Stankiewicz E. Pabisek M. Słoński B. Rodzynkiewicz A. Matuszak

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ISBN: 978-83-7242-684-0

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Prof. Z. Waszczyszyn's CV

PROFESSOR ZENON WASZCZYSZYN BRIEF SCIENTIFIC CURRICULUM VITAE

Professor Zenon Waszczyszyn is a scientist with unique achievements and wide recognition in Poland and abroad. He has a record of 48 year career and service at the Cracow University of Technology (CUT). He is a full member of the Polish Academy of Sciences (PAS), an active member of the Polish Academy of Sciences and Arts (PASA) in Cracow, and Doctor *honoris causa* of Budapest University of Technology and Economics (BUTE).

Professor Zenon Waszczyszyn was born in Lwów, Poland (now Ukraine) on July 12, 1935. He graduated with MSc degree in Civil Engineering at Cracow University of Technology in 1956. In years 1957-1962 he worked at a design office in Cracow and became a licenced engineer. In 1959 he was employed as an assistant at the Chair of Strength of Materials and Structural Statics of CUT. Under the supervision of Prof. M. Życzkowski he embarked on the research into restricted support movability of beams in bending. His dissertation on the subject resulted in the PhD degree with honours in 1964.



He then embarked on scientific cooperation with Prof. A. Sawczuk from the Institute for Fundamental Technological Research of PAS (IPPT PAN) in Warsaw. On the basis of dissertation on finite displacements of elastic-plastic plates and shells of revolution he was awarded the DSc degree (called in Poland *habilitacja*) in 1970, thus becoming associate professor. In years 1973-78 he was the Head of the Computer Center of CUT. He was nominated full professor in 1978 and then till 1992 he was the Head of the Laboratory of Structural Stability and Computational Methods at the Institute of Structural Mechanics of CUT.

In 1989 he got elected a Corresponding Member of the Polish Academy of Sciences and in 1990 he became a Full Member of the Polish Academy of Sciences and Arts (the oldest Polish Academy of Sciences reactivated in Cracow in 1989). In years 1991-93 he served as Vice-Rector for University Staff and International Relations of CUT.

In 1992 he initiated the Institute of Computer Methods in Civil Engineering, where he was the Head of the Chair of Computational Structural Mechanics until 2005 and also the head of the Institute in years 1997-2005. He then worked for 9 years as full professor at the Chair of Structural Mechanics of Rzeszów University of Technology.

In years 2007-10 Professor Waszczyszyn was Vice-President of the Cracow Branch of PAS. In 2011 he was elected vice-president of Department III Math-Phys-Chem of PASA. In 2012 he was elected the head of the Commission of Technical Sciences of PASA. Last year he finished his service in Rzeszów and at present he divides his time between the two Academies he is involved in, Cracow University of

Technology, where he cooperates mainly with Prof. E. Pabisek and Dr M. Słoński, and AGH University of Science and Technology, where he cooperates with Prof. T. Uhl.

In his career Professor Z. Waszczyszyn has had several occasions to work abroad, first at the Tbilisi University, Georgia in 1966, then at the Lomonosov University, Moscow in 1969/70 and at Politecnico di Milano in 1976/77. Then, with DAAD scholarship, he visited German technical universities in Munich, Hanover, Stuttgart and Bochum in 1982. In 1987 he won a competition for the position of visiting professor at Aerospace Faculty of the Delft University of Technology (TU Delft), the Netherlands, and spent a year there doing research and lecturing on computational plasticity. Lib lecture notes on *Computational Methods and Plasticity* were published in 1989 as TU Delft Report LR-583 (214 pages). Later he lectured on various occasions at University Clermont-Ferrand II, TU Delft, Heriot-Watt University, Edinburgh, Universita di Firenze, BUTE i TU Wien.

Professor Waszczyszyn's research activity was primarily related to the modelling and analysis of problems of nonlinear mechanics of structures and materials. His interest in the theory of elasticity and plasticity and, especially, in nonlinear stability of plates and shells opened for him the door to the cooperation with scientists in mechanical and aerospace engineering. He enjoyed long term cooperation with Prof. G. Maier from Politecnico di Milano, Prof. S. Kaliszky from BUTE, Prof. J. Arbocz from Delft University of Technology, Prof. H. Mang from Vienna University of Technology, Prof. V. Zarudsky from the Institute of Mechanics of the Ukrainian Academy of Sciences in Kiev and Prof. U. Lepik from Tartu University, Estonia.

He concluded his activity in the field of structural stability by publishing the book on *Stability of Structures and Finite Element Methods*, Elsevier, Amsterdam, 1994 (468 pages), co-authored by his associates Cz. Cichoń and M. Radwańska, and co-authoring and editing volume 3 of the book in Polish *Mechanika budowli*. *Ujecie komputerowe* (Structural mechanics. Computational approach), Arkady, Warsaw, 1995 (327 pages).



Around his 60th birthday he decided to change the focus of his interests and started investigating the applications of artificial intelligence methods in civil engineering. Initially, he was interested in deterministic artificial neural networks (ANN). Next, he extended the scope of his work to hybrid systems combining neural networks with the finite element method, fuzzy sets and Bayesian inference concept. Such approaches are especially suitable for efficient inverse analyses of engineering problems, including parameter identification. Recently, he has also been interested in so-called health monitoring of structures, and in particular in the assessment of their resistance to random imperfections.

Prof. Waszczyszyn's research was strongly related to the standing Seminar on Applications of Artificial Neural Networks and Soft Computing in Civil Engineering, which has been organized under his supervision at CUT since 1997. The Seminar gathered up to 30 participants from several Polish universities. As

a result some research teams were formed to introduce the applications of soft methods in the research projects developed at the participants' universities. With time, this joint effort became the second scientific school of Professor Waszczyszyn, after the school of structural stability and numerical methods he had founded in the eighties.

In 2001 Professor Waszczyszyn won a so-called professor's subsidy of the Foundation for Polish Science, awarded every year to 15 prominent scientists. It helped him to develop the research on the applications of ANN in civil engineering and support financially the research of 5 young investigators. The results of the research were presented at numerous scientific conferences and, most importantly, in his lectures delivered at the following CISM Advanced Schools in Udine, Italy (two of which he coordinated): *Neural Networks in Mechanics of Structures and Materials* in 1998, *Parameter Identification of Materials and Structures* in 2003 and Advances of Soft Computing in Engineering in 2007. Two valuable books edited by Professor Waszczyszyn were published by Springer after the courses: *Neural Networks in the Analysis and Design of Structures*, CISM Courses and Lectures, vol. 404, Springer, Wien - New York, 1999 (307 pages) and Advances in Soft Computing in Engineering, CISM Courses and Lectures, vol. 512, Springer, Wien - New York, 2010 (336 pages).



Prof. Waszczyszyn directed many research projects related to modelling and computer simulation of materials and structures. He was the principal investigator of 23 Polish grants in the field of mechanics and civil engineering. In his career he supervised fifteen doctoral projects, including seven completed with honours. From among his associates three obtained professor nominations (Cz. Cichoń, K. Kuźniar and L. Ziemiański), and six scientists got the DSc degree (W. Łakota, B. Miller, J. Pamin, E. Pabisek, E. Pieczara, M. Radwańska). He also was the initiator of granting *honoris causa* doctorates of CUT to renowned professors Herbert Mang from TU Wien and Michał Kleiber from IPPT PAN.

Altogether, he is an author and co-author of over 270 publications, including 7 books/monographs and 6 textbooks, over 170 original papers published in scientific journals, several chapters in books, stateof-the-art papers and published plenary and keynote lectures. The results of his research were presented at a multitude of scientific congresses, symposia and conferences. He reviewed over 70 PhD and DSc dissertations and served as a member of examination commission at TU Delft, Herriot-Watt University Edinburgh, TU Helsinki and TU Wien. He is a member of editorial boards of several scientific journals.

He has always paid a lot of attention to the educational problems related to graduate, post graduate and PhD courses. In years 1992-99 he was a member of the SEFI (Societé Européenne pour la Formation des Ingénieurs) Curriculum Development Working Group. In this body he focused on the necessity of introduction of humanities into engineering curricula. In 1992 he was elected an expert of the Polish Ministry of Higher Education in the field of Civil Engineering and has served in this capacity since then. In the 90s he coordinated 2 European TEMPUS projects.

Starting in 1987, at first together with Prof. S. Kaliszky from the Department of Mechanics, BUTE, he co-organized 14 bilateral Budapest-Cracow Inter-Institute Seminars on Numerical Analysis of Structures, mainly devoted to the discussion of the doctoral projects of young scientists. In 2000 Prof. H. Mang and his team from the Institute for Strength of Materials, TU Wien, joined the initiative and the Seminars are continued in a trilateral form under the name Inter-Institute Seminar for Young Researchers. In years 1980, 1985 and 1990, within his activity as the chairman of the Section of Structural Mechanics of the Committee for Civil Engineering of PAS, Prof. Waszczyszyn organized schools of structural stability, and in 1996 a Summer Course on Mechanics of Concrete.

Prof. Waszczyszyn's scientific and educational activity has been well appreciated. In 1997 he was elected SEFI Fellow. In 2001 he was distinguished by the title of Doctor *honoris causa* of Budapest University of Technology and Economics. In 2006 he received the title of IACM Fellow from the International Association for Computational Mechanics. As one of the founders of the Polish Association for Computational Mechanics. As one of the Zienkiewicz Medal of PACM for all his activity in 2006. From among numerous Polish honours the F. Jasiński award of Branch IV (Technical Sciences) of PAS in 1972 and the award of the Minister for National Education and Sport for all achievements are worth mentioning.

He was the chairman of the 2nd European Conference for Computational Mechanics, ECCM, Cracow 2001, and of the International Symposium on Neural Networks and Soft Computing in Structural Engineering NNSC, Cracow 2005. More recently, he co-organized with Professors T. Burczyński and Z. Mróz a series of ECCOMAS thematic conferences on Inverse Problems in Mechanics of Structures and Materials IPM, Rzeszów 2009, 2011 and 2013. These conferences focused on advanced mathematical methods for modelling and analysis of inverse problems in mechanics, intelligent computing strategies, structural health measurement and material parameter identification. These are the fields of the current research of Professor Zenon Waszczyszyn.

Abstracts

MULTISCALE STRUCTURAL ANALYSIS

Bernhard Pichler^{*}, Mohammad Irfan-ul-Hassan, Roland Reihsner, Markus Königsberger, Christian Hellmich, and Herbert A. Mang

> Institute for Mechanics of Materials and Structures Vienna University of Technology (TU Wien), Austria E-mail: Bernhard.Pichler@tuwien.ac.at

KEYWORDS: Hong Kong - Zhuhai - Macau Bridge, innovative material testing, multiscale material modeling

The Hong Kong – Zhuhai – Macau Bridge [1] is one of the most ambitious construction projects of the 21^{th} century. The traffic link exhibits a total length of about 50 km. It consists of two bridges and one central 6.7 km long *immersed tunnel*, see Fig. 1. The latter consists of pre-fabricated reinforced concrete segments with a length of 22.5 m. Eight segments are connected – on a nearby island – to form 180 m long elements of the tunnel. 33 elements are shipped to the construction site. There, they a lowered into a trench at the bottom of the sea, where they are connected in order to form the continuous tunnel. The designed service lifetime of the structure amounts to 120 years.

Designing the immersed tunnel based on currently available international standards was a great challenge for Chinese engineers. This raises the question whether or not an added value can be achieved by involving modern multiscale material models for concrete in the design calculations. Such models are nowadays emerging in the pre-normalization domain. The assessment of the possible added value is the topic of a research project [2] which was started on September 1, 2015, at TU Wien's Institute for Mechanics of Materials and Structures. Four PhD-projects cover the following topics: (i) tunnel safety in case of a fire, (ii) tunnel safety in case of an earthquake, (iii) reliability of force-transfer between neighboring tunnel elements via so-called shear keys, and (iv) mastering the surface cracking risk of the prefabricated reinforced concrete segments at very early ages. In all four mentioned research projects, the same research concept will be implemented.

Figure 1: Immersed tunnel of Hong Kong – Zhuhai – Macau Bridge, after [2]

This research concept is exemplarily shown in the present contribution. It leads from innovative material testing, via modern multiscale material modeling, to the sought multiscale structural analyses. Starting point is a new test protocol providing access to the evolutions of elastic stiffness and creep properties of hydrating cement pastes [3]. Multiscale material modeling allows for top-down identification of "universal" creep properties (= material *constants*) of cement hydrates, see the smallest scale in Fig. 1. Predictive capabilities of the developed multiscale creep model are checked quantitatively, and they turn out to be satisfactory. Therefore, the multiscale material model is capable of providing a reliable description of the creep properties of the concrete used for the immersed tunnel, and this is expected to increase the reliability of structural simulations.

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- [2] Mang H. A. and Pichler B., *Bridging the Gap by Means of Multiscale Structural Analyses*, Fundamental-research project funded by the Austrian Science Fund (FWF). Website: http://www.fwf.ac.at/en/
- [3] Irfan-ul-Hassan M., Pichler B., Reihsner R., and Hellmich Ch., *Elastic and creep properties of young cement paste, as determined from hourly repeated minutes-long quasi-static tests*, submitted to Cement & Concrete Research. Under revision.

DISCRETE ELEMENT ANALYSIS OF SKEWED MASONRY ARCHES

Tamás FORGÁCS^{*}, Vasilis SARHOSIS[•], Katalin BAGI^{*}

* Dept. of Structural Mechanics, Budapest University of Technology and Economics * University of Cardiff E-mail: tamasforgacs@hotmail.com ; SarhosisV@cardiff.ac.uk ; kbagi@mail.bme.hu

KEYWORDS: oblique arches, stereotomy, 3DEC

Masonry arch bridges are inherent elements of Europe's transportation system. Many of these bridges have spans with a varying amount of skew. Most of them are well over 100 years old and are supporting traffic loads many times above those originally designed, but the increasing traffic loads may endanger their structural integrity so the need arises to understand the mechanical behaviour in order to inform repair and strengthening options.

There are three main construction methods mostly used in such bridges. The differences in geometry lead to differences in strength and stiffness.

The study to be presented investigates the mechanical behaviour of single span masonry arches. The analysed construction methods were the so-called false skew arch, the helicoidal and the logarithmic method. The three-dimensional computational software 3DEC based on the discrete element method was used: this software allows for the simulation of frictional sliding and separation of neighbouring stone blocks. The behaviour of the structures was simulated under gravity. Types of failure modes, stress levels, shear displacements at the joints will be compared and discussed in the presentation. The minimum necessary thickness which can resist the self-weight was also determined.

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- [2] Sarhosis, V., Oliveira, D.V, Lemos, J., Sheng, Y., Lourenco, P. B. The behaviour of single span stone masonry skew arch. 7th International Conference on Arch Bridges, Split, Croatia 2013

THREE-DIMENSIONAL FE SIMULATION OF CORRODED BEAM IN 3-POINT BENDING

Magdalena German, Jerzy Pamin

Institute for Computational Civil Engineering Cracow University of Technology, Poland E-mail: mgerman@L5.pk.edu.pl, jpamin@L5.pk.edu.pl

KEYWORDS: corrosion propagation, concrete simulation, cracking

The paper presents a 3D model of a reinforced concrete element with rust introduced as interphase between steel and concrete. The analysis brings information about the loss of bond between concrete and steel due to corrosion phenomenon. The main effects of corrosion on bond are the reduction in the confinement of the rebars caused by the opening of longitudinal cracks along the reinforcement and the reduction of friction due to the processes occurring at the steel-concrete interface [1], [2], [3]. The paper presents 3D simulations of an RC beam model in 3-point bending test. The model considers two materials – concrete described using plasticity-based damage model available in Abaqus [5] and steel modelled as an elastic-plastic material with hardening. Rust is introduced as an interface between steel and concrete, whose response is defined in terms of traction versus separation. The simulation is performed using Abaqus, a tool based on the Finite Element Method. The model has been made in accordance with the experiment performed by [4]. During the analysis the reduction of load-carrying capacity is controlled. Moreover, the simulation is going to demonstrate whether the loading sequence (corrosion and deflection) has the influence on beam response. Some representative results, calculated for corrosion level 5% and deflection in the middle of the span 10mm, are presented in Fig. 1.

Figure 1: Distribution of equivalent plastic tensile strain (PEEQT): (a) – isometric view, (b) – cross-section view.

- G.J. Al-Sulaimani, M. Kaleemullah, I.A. Basunbul and Rasheeduzzafar. Influence of corrosion and cracking on bond behavior and strength of reinforced concrete members. *ACI Structural Journal*, 87:220–231, 1990.
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STRUCTURAL BEHAVIOR OF CONCRETE HINGES EXPERIMENTAL RESULTS AND MECHANICAL MODEL

Thomas Schlappal*, Michael Schweigler, Bernhard Pichler

Institute for Mechanics of Materials and Structures Vienna University of Technology (TU Wien), Austria E-mail: thomas.schlappal@tuwien.ac.at

KEYWORDS: tensile cracking of concrete, stress triaxiality, steel reinforcements, digital image correlation

Concrete hinges are nowadays experiencing a renaissance in practical civil engineering. In order to further improve the design guidelines by Leonhardt [1], we here combine (i) lab-scale experiments, where the structural behavior of concrete hinges under eccentric compression is measured by means of Linear Variable Differential Transducers (LVDTs) as well as by state-of-the-art contact-free deformation measurements (Digital Image Correlation) and (ii) an engineering mechanics model accounting for key features of the structural behavior, involving cracking of concrete under tension as well as compression under markedly triaxial stress conditions.

The investigated concrete hinges exhibit a width of 25 cm, a height of 35 cm, a depth of 40 cm, and a neck width of 7.5 cm. Only three pairs of crossed steel rebars connect the two steel reinforcement cages of the top and bottom parts of the concrete hinge. As for concrete, C30/37 with a maximum aggregate size of 16 mm is used. 28 days after production, the structure is subjected to compressive line loads acting with an eccentricity of 2.5 cm relative to the plane of symmetry (Figure 1).

Figure 1: Concrete hinge in testing machine: (a) top view, (b) side view

A digital image correlation system Dantec Dynamics Q 400 is used to measure surface deformations in the immediate vicinity of the neck. In order to capture tensile crack propagation, both the front and the back surface of the concrete hinge are monitored with two cameras each. To this end, both surfaces are painted, prior to the test, with fine speckle patterns. In order to quantify the relative rotation angle across the neck of the concrete hinge, ten Linear Variable Differential Transducers of type HBM W1/2mm-T are used (Figure 1).

As regards modelling, we consider that a concrete hinge represents a neck in a reinforced concrete member subjected to both an axial force and bending. The neck results in stress concentrations which render the Euler-Bernoulli hypothesis of classical beam theory inapplicable. As a remedy, we use the Finite Element Method to compute the elastic stress distributions in the cross-section of the neck. This spatially non-linear stress distribution is valid until the tensile strength of concrete is reached at the edge of the neck. Beyond that, we use a tensile cracking law to describe the structural behavior in terms of the relation between rotation angle and external loading. The engineering mechanics model extends Leonhardt's triangular stress field [1] to a more realistic stress distribution. This way, we aim at supporting future structural design of concrete hinges.

REFERENCES

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A HYBRID COMPUTATIONAL SYSTEM FOR IDENTIFICATION OF ELASTIC THIN PLATES PARAMETERS USING LAMB WAVES PROPAGATION AND ARTIFICIAL NEURAL NETWORKS

Zenon Waszczyszyn¹, Ewa Pabisek¹, Piotr Nazarko², Łukasz Ambozinski³, Piotr Paćko³, Tadeusz Uhl³

¹ Institute for Computational Civil Engineering Cracow University of Technology, Poland e-mail: {zenwa, epabisek}@L5.pk.edu.pl ² Chair of Structural Mechanics Rzeszów University of Technology, Poland e-mail: pnazarko@prz.edu.pl

³ Chair of Mechatronics and Robotics AGH University of Science and Technology, Poland emails: {pawel.packo, ambrozine, tuhl}@agh.edu.pl}

KEYWORDS: Hybrid Computational System, Lamb Waves, Artificial Neural Networks

1. INTRODUCTORY REMARKS

A Hybrid Computational System (HCS) is presented in the paper. The system consists of two approaches, i.e. direct analysis and reverse analysis. Hybrid character of HCS corresponds to the application of hard and soft computing. HCS are applied in Structure Health Monitoring/Measurement (SHM) as a new research tool in science and technology [1]. Due to monitoring or measurement on real structures, SHM can reflect the actual state (health) of structures in order to their control and warning against failure or dangerous events, without application of destructive methods for structure examination.

It was proved in the analyses of many engineering problems, that the Artificial Neural Networks (ANNs) can be efficiently applied in the analysis of reverse (identification) problems [2–4]. This feature is complementary to direct (simulation) features of other basic numerical methods, e.g. FEM or a number of experimental laboratory tests or measurements on real structures.

In the area of direct analysis, basic components of HCM are experimental data related to propagation of ultrasonic waves in solid media. A special kind of theses waves, called Lamb Waves (LWs), was selected as a good non-destructive method. These waves are guided and propagated in thin plates along a comparatively long distance, see [1, 5].

From the view point of SHM especially interesting is formulation of HCS (HCS) from three components, i.e. data from tests or measurement on real structures, their processing and neural identification. Thus, HCS can play the basic role in the formulation of an original non-destructive method.

In the paper we mention in short our increasing interest for formulating of HCS in the frame of the activity of RGNN and SemNN, see below Point 2. Then some basics on Lamb waves and corresponding physical equations are shown. Special attention is paid to discussion on computational aspects of the title problem in the frame of so-called Essential Four Step algorithm, cf. [1], and its modifications. They correspond to the main novelty of our paper depending on separation of the direct and reverse analysis. Selected results from two case studies are discussed in order to point out the analytical and numerical problems related to the identification of material properties of elastic, isotropic thin plates and unidirectional reinforced composite lamina.

2. DEVELOPMENT OF HCS IN RGNN AND SEMNN

At the turning of 1995/96, an informal Research Group on Applications of NN in Civil Engineering (called RGNN for short) was organized at the Institute on Computer Methods in Civil Engineering of Cracow UT. The standing Seminar (SemNN) was also organized as accompanying part of the RGNN. This activity attracted participants from eight Polish TUs, see references in [3, 4]. During of more than fifteen years of the RGNN and SemNN activity, a great attention was focused to the analysis of inverse problems since just in such topics the ANNs occur to be a new, numerically efficient tool.

The corresponding research and engineering applications need formulation of different HCSs. They were developing by RGNN participants, especially from Cracow, Rzeszów and Zielona Góra TUs., see [3–4]. It is worth mentioning PhD theses by B. Miller, J. Kaliszuk and P. Nazarko as well as dissertations written for position of Assoc. Prof. by W. Łakota, E. Pabisek and B. Miller. A very fruitful was the time 2013-15 when also a new HCS within grant of the Polish Science Foundation was under development. Just basing on this grant, a topic of the presented paper has been written. In this field, the cooperation with Ł. Ambroziński and P. Paćko from the Professor's T. Uhl research group at AGH Kraków was unavailable.

3. NOVELTIES IN THE PROPOSED HCS

3.1 Modification of the Four Essential Steps algorithm

A flow-chart algorithm, discussed in [1], is a scheme of a general algorithm for the identification of damage in thin plates, see Figure 1 taken from[6]. This algorithm has been modified in the presented lecture order to o have a full correspondence to the proposed new HCS. The flow-chart was divided into two Parts A and B. The Part A is completed of three Essential Steps, related to the direct analysis. The Part B, contains an extended Essential Step IV in which an ANN is placed for the reverse analysis of composite plates.

Figure 1: Four essential steps for identification of plate parameters, applying Lamb wave measurement technique and identification procedures in step IV: CL –Classical approach, ANN – artificial neural network

In case of isotropic material, it has been assumed in [6] that identified plates are homogenous, of constant thickness d = 2h and density ρ , made of elastic material with two mechanical parameters E and v.

The main attribute of the problem is the Lamb dispersion curve (LDC). It is a dependent variable k (f | **spar**): where: k – wave number measured, f [MHz] – frequency of vibrations, **spar** – selected set of independent variables.

The separation of Parts A and B leads to "numerical on line" approach. Instead of the classical approach (CL) minimizing the "distance" between the experimental and Lamb DC $\|\tilde{k}_{exp} - \tilde{k}_{exp}\|$, only one substitution approach is applied. This means that the values of the internal parameters $\{\alpha_j\}$, computed as outputs of Part A are inputs to the 'off line' trained ANN in Part B. In such a way, the commonly used classical approach, which needs the application of different numerical methods, e.g. FEM BEM FDM and other special methods, can be eliminated.

3.2. Pattern generation for the trained 'off line' ANN

A novelty in [6] corresponds to the introduction of approximate dimensionless LDC equations, derived by Armikulova in her M. Sci. [7], for the computation of points in the (k, f) space of isotropic plates. In the presented paper we also derived basic equations by means on dimensionless variables, which make possible to accelerate significantly the generation of patterns during the 'off line training' of the ANN in the Stage B.

In case of composite lamina a very general and extended procedures EFIT/LISA/FDM (Elastodynamic Finite Integration Technique/ Local Interaction Simulation Approach/ Finite Difference Method) were applied in [8]. These procedures are numerically very "costly" (needs a great number of operation) so quite recently they have been changing into numerically very efficient procedures supported on ALIMEQ (Algebraic Implicit Equations).

The proposed approaches were examined by some case studies, using experimental tests, as well as pseudoexperimental and noisy data.

3.3. Identification problems in composite plates - some remarks and conclusions

i) The isotropic material mentioned above has two independent material mechanical parameters E and v. In case of the hexagonal orthotropic, five independent parameters C_{ij} have to be identified Such a model of material well corresponds to the uniaxially reinforced composite lamina.

ii) It was proved in paper [8], which is a generalization of [6], that the separation of Parts A and B in a corresponding HCS is also possible for anisotropic materials.

iii) One of the main problem are very different and comparatively small values of stiffness parameters in case of anisotropic materials. For instance, the stiffness corresponding to tension of composite reinforcement $C_{11} \approx$ 130 MP is much more lower than the stiffness $C_{44} \approx 4.0$ MPa for shear. Moreover, such low values of stiffness are measured with about 20% accuracy, see [5].

iv) Another important question is the sensitivity of stiffness to variations of parameters. The sensitivity analysis enables us to select vibration modes for the generation of patterns for the ANNs training 'off line', see Figure 2.

v) Unlike the isotropic material, where only one LDC can be used for the mode A_0 , see [4], in case of the above mentioned composite material we have to apply two modes A_0 and B_0 , see Figure 2. The uniqueness of the stiffness evaluation demands three independent guided directions, i.s. with inclinations from the direction of reinforcements $\alpha = 0^\circ$, 90° and 45

vi) Now we are interested in development of research on application of the HCS to the analysis of composite plies composed of unidirectional reinforced lamina.

Figure 2. Sensitivity of modes A_0 and S_0 to changes of plate stiffnesses C_{11} and C_{44}

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FROM PNEUMATIC BEAMS TO WOVEN BASKETS: ADVENTURES ON THE SURFACE OF CUBOIDS

Flórián Kovács, Krisztián Hincz, András Lengyel, Tibor Tarnai

Department of Structural Mechanics Budapest University of Technology and Economics, Hungary E-mail: kovacs.florian@epito.bme.hu

KEYWORDS: cube, pneumatic beam, convex polyhedron, geodesic, inextensional deformation, weaving

This talk reviews parallel investigations that have to do in common with closed convex polyhedral surfaces. These analyses extend, at the same time, from applied engineering to fundamental research on structural topology.

Pneumatic beams were investigated in [1] by both analytical and experimental methods, with the principal aim of finding the ultimate load in a four-point flexural test. Real-scale experiments were also tested by FEM-analysis of fabrics with triangular elements. Both analyses confirmed that, in spite of that was commonly considered earlier, such beams have a considerable load bearing capacity even beyond the limit when lateral warping starts on the top.

Another pneumatic structure is an inflated cube with inextensional deformation. In this research, the main question has been inspired by Alexandrov's theorem [2] on unique convex realization of solids. In this view, a cube of increased volume is surely nonconvex but it is an open question in what amount the original volume can be increased by inextensional deformations. In this research [3], a larger volume than described ever in literature is found. At the same time, there exists a theoretical limit for the maximum volume whose geometric realization is still unknown.

Cubic surfaces are not only examined from the aspect of their volume enclosed but also from aspects of topology of systems on the surface itself. If a surface of a cube is covered by some strands of equal width, crossing at a right angle, they can be arranged in an alternate twofold weaving consisting of a finite number of closed strands following closed geodesics on the same surface. One possible question is about the number of strands at different obliqueness of the pattern on the surface [4], but if the analysis is extended to polyhedra that are cube only combinatorially (cuboids), it should also be asked what conditions should their *geometry* obey and how the network of such weavable solids can be derived.

If a rectangular box is bound by some closed loops of strings (as is commonly happens to gift boxes, etc.), the analysis of its possible geometries can easily be related to the previous research. In addition, however, it can be asked whether or not a loop or a system of connected loops is stable (i.e., the box cannot be freed from cables without cuts or stretching). For the sake of simplicity, no friction is assumed; and a search for both the minimal number as well as minimal total length of such stable bindings was performed [5]. Among others, it could be concluded that minimum number of loops on such a box is two, but it is associated with a skew network of cables.

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A STRESS-BASED APPROACH IN FREE MATERIAL DESIGN

Sławomir Czarnecki

Department of Structural Mechanics and Computer Aided Engineering Warsaw University of Technology, Poland E-mail: s.czarnecki@il.pw.edu.pl

KEYWORDS: topology optimization, free material design, anisotropic and isotropic elasticity, minimum compliance problem, stress-based formulation

The structures made of a homogeneous material are non-optimal even if their shape is rationally designed by a certain optimization technique. Thus, replacing the homogeneous material by a non-homogeneous one with rationally designed material properties seems to be a natural generalization of the shape optimization (see e.g. [1-4]). In the paper, the problem of finding an optimal distribution of elastic moduli within a given domain Ω occupied by a non-homogeneous, elastic body to make its compliance minimal is presented. To make the problem mathematically well-posed, an isoperimetric condition (interpreted e.g. as a condition limiting the technological cost) is imposed on the integral of the trace of the Hooke elasticity tensor $\mathbf{C}(x)$, $x \in \Omega$. In the case of three dimensional, non-homogeneous, anisotropic elastic body, the six Kelvin moduli $\lambda_K(x)$ and six eigenstates $\mathbf{\omega}_K(x)$, (K = 1, 2, ..., 6) defining the spectral decomposition $\mathbf{C} = \sum_{k} \lambda_K \mathbf{\omega}_K \otimes \mathbf{\omega}_K$ of the Hooke tensor \mathbf{C} are the design variables. In the case of non-homogeneous,

isotropic elastic body, the bulk k = k(x) and shear $\mu = \mu(x)$ moduli defining the spectral decomposition

$$\mathbf{C} = 3k \,\mathbf{\Lambda}_1 + 2\mu \,\mathbf{\Lambda}_2, \quad \mathbf{\Lambda}_1 = \frac{\delta_{ij} \delta_{kl}}{3} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l, \quad \mathbf{\Lambda}_2 = \frac{\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}}{2} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l - \mathbf{\Lambda}_1 \quad \text{of the isotropic}$$

Hooke tensor **C** are the only two unknown scalar fields. Let us denote the vector of surface tractions applied to the free boundary $\Gamma_1 \subset \partial \Omega$ of the body Ω and the vector of unknown displacements by **t** and **u**, respectively. In the present paper it has been shown that the original problem $\min_{\text{admissible } C} \int_{\Gamma} \mathbf{t} \cdot \mathbf{u}(\mathbf{C}) da$ can be

reduced to a new problem $\min_{\text{statically admissible } \tau} \int_{\Omega} \| \mathbf{\tau} \| dx$ of minimization of the integral of the stress tensor norm

 $\|| \tau \||$ over statically admissible stresses τ (norm definition $\|| \cdot \||$ is different for anisotropic and isotropic optimization). The formulae for optimal Kelvin's moduli and eigenstates or bulk and shear moduli are analytically determined by the stress field $\tau^*(x)$ being the minimizer of this problem.

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ROTATIONAL STIFFNESS OF A NODE AS AN UPDATED PARAMETER OF A NUMERICAL MODEL OF THE TWO-STOREY PORTAL FRAME

Dominika Ziaja, Karol Lech, Bartosz Miller

Department of Structural Mechanics Rzeszow University of Technology, Poland E-mail: dziaja@prz.edu.pl, 122206@sd.prz.edu.pl, Bartosz.Miller@prz.edu.pl

KEYWORDS: structural health monitoring, modeling, genetic algorithms, artificial neural networks

Subject of the article is associated with FE model updating treated herein as the first step of Structural Health Monitoring procedure [1]. The purpose is to find rotational stiffness of six nodes of two-storey steel frame. These stiffnesses (two for column footings: k_1 - k_2 and four for beam-to-column connections: k_3 - k_6) were sole parameters being updated in plane FEM model. Data for model updating come from dynamic measurements performed on a laboratory model of considered frame. The paper describes the laboratory model, used equipment as well as measurement set-up.

Beam-to-column connection was deeply examined. For this connection both shell and solid FEM model were created and analysed. Three methods of obtaining the node rotation stiffenss were applied. The results allowed to narrow down the range of stiffness between 800 and 1650 kNm/rad taken into account in the updating process what improved updating process accuracy. Three types of tools were applied: Genetic Algorithms (GA)[2], Artificial Neural Networks (ANN)[3] and Gaussian Processes (GP)[4]. The results are collected bellow.

All methods allowed to update the FEM model with satisfactory accuracy, but the time consuming was different for each of them. The fastest one was AG. The differences between results, obtained using three analysed tools, were rather symbolic. Values of rotational stiffness show, how important is length of column below and above the node. The upper connections have smaller stiffness, despite the part of column, protruding above the end-plate.

		AG		SSN		GP	
Rotational stiffness [kNm/rad]	<i>k</i> ₁	244.0		88.5		153.9	
	k_2	97.0		212.7		104.0	
	<i>k</i> ₃	x ₃ 1094.4		1248.4		1155.6	
	k_4	1094.9		1306.0		1155.8	
	k_5	800.3		1010.4		964.5	
	<i>k</i> ₆	813.9		845.4		854.4	
Physical model f_{phys}		fnum	MAC	fnum	MAC	fnum	MAC
[Hz]		[Hz]	[-]	[Hz]	[-]	[Hz]	[-]
25.7		25.8	0.97	25.8	0.97	25.2	0.97
113.9		115.3	0.85	115.5	0.82	114.8	0.83
143.9		142.7	0.90	144.1	0.90	143.8	0.89
160.7		159.5	0.95	161.9	0.95	160.4	0.94

The comparison of results of model updating depending on used methods

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DUAL-SCALE POROSITY ANALYSIS OF BRICKS WITH DIFFERENT PORE-FORMING AGENTS AND CONCENTRATIONS

Hawraa Kariem, Josef Füssl, Christian Hellmich

Institute for Mechanics of Materials and Structures Vienna University of Technology, Austria E-mail: hawraa.kariem@tuwien.ac.at, josef.fuessl@tuwien.ac.at, christian.hellmich@tuwien.ac.at

KEYWORDS: masonry brick, porosity, micro-CT

Masonry brick is one of the most commonly used building materials and its thermal conductivity plays an important role in its energy efficiency. Since thermal conductivity depends on the porosity of the material, the latter needs to be quantified first. Using micro-CT images the dual-scale porosity of bricks fired at 880 °C with different pore-forming agents and different concentrations was determined. The different pore-forming agents included expanded polystyrene (EPS) with a concentration of 10% and 20%, paper sludge, and sawdust in three different concentrations: 10%, 20% and 40%. For reference purposes, brick samples made of pure clay without pore-forming agents fired at 880 °C and 1100 °C were also evaluated.

The open source image-processing and analyzing software ImageJ [1] was used to segment the micro-CT images into macropores and a solid phase via the Otsu-algorithm [2]. The volume, the volume distribution, the minimum bounding box and the orientation of the macropores were determined using different plug-ins of ImageJ. The linear relationship between the grey values and the photon energy was statistically analyzed and knowing the chemical composition of the brick matrix the microporosity of the solid phase was determined [3]. Brick samples with EPS have the biggest macropores, whereas samples with sawdust have the biggest volume fraction of macropores in comparison to EPS and paper sludge at the same concentration. In contrast, samples with paper sludge have no detectable macroporosity and the highest microporosity. EPS did not influence the microporosity, since the samples with EPS have almost the same microporosity as the brick samples made of pure clay without a pore-forming agent. The overall porosity of the brick samples was also determined using the micro-CT images and validated using mercury intrusion porosimetry.

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DUCTILITY OF THE EMBEDMENT BEHAVIOR OF STEEL DOWELS IN WOOD – PERMANENT DEFORMATIONS AND THE INFLUENCE OF REINFORCEMENTS

Wolfgang Lederer, Thomas K. Bader and Josef Eberhardsteiner

Institute for Mechanics of Materials and Structures Vienna University of Technology (TU Wien), Austria e-mail: wolfgang.lederer@tuwien.ac.at

KEYWORDS: wood, dowel connections, reinforcement, digital image correlation, MicroCT

The performance of connections in timber structures is of particular importance, as these parts are generally weaker than the connected timber elements and, therefore, may govern failure of the entire structure. The load-transfer in steel dowel connections – the most common type of connectors in engineered timber structures – is based on the so-called embedment behavior of wood, which will be thoroughly discussed in this contribution. Experiments were focused but not limited to the ductile behavior of dowel connections with the aim to quantify the local material behavior of wood as well as overall connection properties. Ductility is one of the beneficial characteristics of dowel connections (see e.g. [1]) and can be ensured by preventing brittle failure with reinforcement measures (see e.g. [2]). Thus, experiments aimed at quantifying the influence of reinforcements on the overall behavior of dowel connections.

On the local (material) scale close to the wood-dowel interface, X-ray micro-computed tomography (MicroCT) was used to examine the development of plastic deformations in wood during embedment tests parallel and perpendicular to the grain [3]. Furthermore, embedment tests with different types of reinforcements were performed in order to measure the embedment strength and deformation characteristics. For this purpose, a full-field optical measurement system, based on digital image correlation (DIC) was used to study surface strain concentrations as well as the initiation and propagation of cracks.

The close vicinity of the dowel hole was scanned by means of MicroCT after unloading at three typical load stages, namely the end of the dowel adjustment, at the quasi-elastic limit and at a dowel displacement of 5 mm. Intensity histograms, resulting from these scans, confirmed that the main plastic deformations are a result of microstructural failure of wood cells that occurred after the quasi-elastic limit was reached. Additionally, it could be demonstrated that the anatomic structure of wood and its orientation affected the deformation to a great extent. The surface strain fields from the DIC-system as well as the density distribution around the dowel hole provided new insight into the development of plastic deformations and the distribution of tensile stresses perpendicular to the grain.

Embedment tests with different types of reinforcements, namely different types, numbers and positions of screws as well as different engineered wood products and nail plates, verified the potential of these measures to ensure a ductile behavior of dowel connections. The active contribution of some reinforcements, leading to an increase of embedment strength could be demonstrated as well. These results may stimulate discussions related to the development of modelling approaches for dowel connections. At the same time, the test results contribute to the long-term goals of this research, which are an efficient use of reinforcements as well as an improved design of dowel connections in timber constructions.

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A MULTISCALE SYSTEMS BIOLOGY APPROACH FOR COMPUTER SIMULATION-BASED PREDICTION OF BONE REMODELING

Maria-Ioana Pastrama ^{*1}, Stefan Scheiner¹, Peter Pivonka² and Christian Hellmich¹

¹Institute for Mechanics of Materials and Structures ,Vienna University of Technology, Karlsplatz 13/202, A-1040 Vienna, Austria

²Australian Institute for Musculoskeletal Science, NorthWest Academic Centre, The University of Melbourne, 176 Furlong Road, St. Albans VIC 3021, Australia

KEYWORDS: bone remodeling, computational systems biology, bone vascular porosity, multiscale poromicromechanics

Bone is a remarkable biological material due to its hierarchical structure, revealing different elements and compositions at different length scales, from trabecular or cortical structures at the macroscopic level, to collagen fibrils and hydroxyapatite crystals at the nanoscale. Another exceptional feature of bone is its ability to continuously renew itself and adapt to changing mechanical loading. Bone remodeling - a process taking place throughout the whole life of vertebrates, allowing for repair of microcracks - involves removal of mature bone by osteoclasts (bone-resorbing cells) and subsequent formation of new bone tissue by osteoblasts (bone-forming cells). These cells respond to biochemical factors such as hormones and local cytokines [1, 2], as well as to mechanical stimuli - all of which are then "translated" into biochemical signals regulating bone remodeling. An imbalance between bone resorption and bone formation can significantly affect the tissue's load-carrying capacity; therefore, the understanding of the complex remodeling process can provide insights into the treatment of several disorders and conditions.

In the current work, a mathematical model derived from a previously published modeling strategy [3] is presented, based on which the dynamics of the bone remodeling process can be accurately predicted. As bone is a macroscopically homogeneous, but microscopically heterogeneous material, it is practical to define a representative volume element (RVE) for describing cell activity: a statistically relevant subvolume of tissue which includes all properties of the macroscopic material. Within such a RVE, bone exhibits double porosity: (i) a vascular porosity, represented by pores with diameters of 10-100 µm, accomodating the blood vessels vascularizing the tissue, and (ii) a lacunar porosity, with pores on the order of 0.1-10 µm in diameter, in which bone-embedded cells (osteocytes) reside. The mathematical model considers the hierarchical nature of bone, and the fact that remodeling takes places in the vascular pore space as a RVE-subspace, where bone cells are located, and where they attach to the pore walls and work in teams to resorb and form new bone. Thus, in the model, biochemical factors and cells are localized in the vascular pore spaces and quantified in terms of vascular concentrations, while mechanical loading is quantified in terms of vascular and lacunar pore pressures. The complex model therefore integrates a systems biology-based mathematical framework, consisting of a series of differential equations describing the evolution of the vascular concentrations of cells and biochemical factors, coupled with a poromicromechanics model of bone. The latter combines a multiply validated, multiscale micromechanics model of bone with classical poroelasticity theory, used to estimate the pore pressures in the vascular and lacunar spaces [4, 5].

Finally, the model is applied for studying the development of the bone composition in the course of mechanical disuse and training-induced bone formation, with the simulation results being in good agreement with related experimental data. This opens new possibilities for the use of computer-based simulations in monitoring extreme remodeling activity, as a consequence of spaceflight or bone-related pathologies.

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^{*}maria-ioana.pastrama@tuwien.ac.at

MODELING OF HETEROGENEOUS MATERIALS USING *hp*-ADAPTIVE FEM AND LOCAL NUMERICAL HOMOGENIZATION

Marek Klimczak, Witold Cecot

Institute for Computational Civil Engineering Cracow University of Technology, Poland E-mail: mklimczak@L5.pk.edu.pl, plcecot@cyf-kr.edu.pl

KEYWORDS: hp-adaptive FEM, multiscale modeling, local numerical homogenization

Motivation of our research is to propose a reliable way of heterogeneous materials modeling. We are interested in a general class of such materials, i.e. with non-periodic structure and inclusions of arbitrary shape. Separation of macro- and microscale is not required.

Despite the constant increase of the computational power, structures made of the aforementioned materials cannot be analyzed using a direct, so-called *brute force*, approach. Even if it was possible, amount of resulting information would be infeasible to be managed effectively.

A variety of homogenization techniques have been developed in order to overcome problems with heterogeneous materials modeling. The most popular approaches are: RVE-based computational homogenization techniques, self consistent schemes, Mori-Tanaka schemes, multiscale FEM, multigrid homogenization.

In our research we take advantage of a novel approach - local numerical homogenization (see [1] and [2]). Its main idea is to solve the main problem using a coarse mesh instead of a fine one. Refinements complying with the heterogeneities are performed locally within every coarse element independently. Thus, the method can be easily parallelized. Effective coarse element stiffness matrix is computed in such a way that the difference between coarse and fine solutions was minimum. Preliminary results performed on uniform meshes confirm the efficiency of the method for linear problems.

In order to generate optimal meshes at two scales of analysis (*macro* and *micro*) we take advantage of the well established automatic *hp*-adaptive FEM (see [1]). Convergence rate for this method is exponential.Adaptation strategy at each iteration step is to find the mesh, which minimizes the error at minimum cost (additional degrees of freedom).

Integration of local numerical homogenization and *hp*-adaptive FEM is quite natural. First, *hp*-adaptation serves as a generator of a coarse mesh. Subsequently, at the level of a single coarse element mesh is refined once again in order to capture the information on the microstructure thoroughly. Using local numerical homogenization effective stiffness matrices are computed for the coarse elements. Finally, the main problem is solved using the coarse mesh.

Numerical results confirm the convergence and reliability of the proposed approach. Adaptivity makes the whole routine more effective. So far, only *h*-adaptivity has been used. Further research effort is to make use of the *hp*-adaptivity and viscoelastic constitutive model.

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HOMOGENIZATION BY THE MULTIGRID TECHNIQUE

Witold Cecot, Marta Oleksy and Michal Krowczynski

Institute for Computational Civil Engineering Cracow University of Technology, Poland E-mail: witek@L5.pk.edu.pl, moleksy@L5.pk.edu.pl, M.Krowczynski@L5.pk.edu.pl

KEYWORDS: heterogeneous materials, algebraic homogenization, error convergence

Direct and sufficiently accurate approximation of solutions to problems with heterogeneous materials usually leads to a huge number of degrees of freedom. Therefore, many homogenization methods have been developed to reduce the computational effort. The objective of the lecture is to give a brief review of the most frequently used homogenization techniques and to present details of the multigrid approach to upscaling process. This algebraic homogenization [6] proved to be an efficient method of approximation for problems with rapidly varying, possibly noncontinuous coefficients as well as with non coherent (porous) domains that may or may not depict a periodic micro structure. Our main contribution is an improved, appropriate for the hierarchical shape functions, intergrid mapping and consequently a fast convergence of both displacements and stresses.

Generally, the multigrid method [3] may be used in two different ways for heterogeneous materials. Either, in a special version accommodated for fast varying material parameters in order to obtain efficiently a direct numerical solution on the most fine grid [1] or as an upscaling method [6] that leads to the so called algebraically homogenized solution on the coarsest mesh. We have shown that the later method is equivalent to the Multiscale FEM (MsFEM) [5], in which special shape functions are constructed, similarly as it was proposed in [2], to resolve the details of material heterogeneities. Typically first order shape functions are used. Only recently [7], the higher order Lagrange type bases were applied, however without any thorough study of convergence.

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PORTABLE TENT STRUCTURE VARIATIONS FOR FLOOD PROTECTION

Róbert Nagy, Zsolt Gáspár

Department of Structural Mechanics Budapest University of Technology and Economics (BUTE), Hungary E-mails: robert.nagy@ mail.bme.hu, ibojtar@mail.bme.hu

KEYWORDS: portable tent, flood protection, sessile drop, elastica equation, nonlinear pendulum, analytic solution

Due to climate change, peak water levels exceed more and more often the height of the existing artificial embankment dams, causing ever increasing difficulties in these flood prone, inhabited areas near regulated rivers.

We present two possible alternatives to laying sandbags, both – being light portable tent structures – are fast and easy not only to transport but to construct as well. In the first an inflated cylinder lifts the top of the textile as the water level rises, while in the second the textile supporting the water pressure of the tide is hanged on a steel framework and is filled with water previously.

Figure 1 – Cross-sections of the two arrangements

We give a numerical solution for the nonlinear system of equations of the compatibility and equilibrium conditions, and also present a complete analytic solution to the boundary value problem of the shape of an inelastic, weightless, prismatic textile, similar to the sessile drop problem. After discussing the mechanical behavior of the structures, we also present different geometries capable of withstanding a certain water height, determined by a software developed for this specific purpose enabling the design for optimum, emphasizing the basic rules of thumb aiding the preliminary conceptual design.

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OPTIMIZATION OF CURVED PLATED STRUCTURES WITH THE FINITE STRIP AND FINITE ELEMENT METHODS

Bence Balogh, János Lógó

Department of Structural Mechanics Budapest University of Technology and Economics E-mail: bbence@eik.bme.hu, logo@ep-mech.me.bme.hu

KEYWORDS: Topology, optimality criteria, finite element method, finite strip method, FEM, FSM.

The need for economical design have been increased during the past decades. In case of structures, optimization means selecting the type of the structure, which fits best the requirements.

The theoretical background of topology optimization was laid down by the Australian scientist Michell, who derived optimality conditions for the optimal topology of lightweight structures in 1904 [1]. The so called optimality criteria methods became popular only in the seventies. This is the period when the first article in the topic of topology optimization was published by Rossow and Taylor [2].

Folded plated structures play a significant role in structural engineering practice, whose cost-effective design can reduce the total costs significantly. A few examples of this type is illustrated in Figure 1.

Figure 1. Examples of folded plated structures

The aim of this study is to compare two available numerical tool for solving PDEs for the optimal design of structures. In the past years numerous methods were developed for topology optimization, from these we have adopted the optimality criteria (OC) approach. The main idea is that we state the optimal conditions which the minimizer has to fulfill at the end of an iterative process. This method however is not a general one, only advantageous in the case of separable problems, but comes with fast speed, easy programming, and a relative insensitivity of computational time to the number of variables. In the paper we suggest a new method for the elimination of a numerical error, the so called 'checkerboard pattern'. In the presented examples we applied one loading case and an elastic material behavior. The cost function is the net weight of the structure and upper bound of the compliance is set as the optimality constraint.

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ADVANCES IN IMPROVING EFFICIENCY OF EVOLUTIONARY ALGORITHMS FOR LARGE OPTIMIZATION PROBLEMS

Janusz Orkisz, Maciej Glowacki

Institute for Computational Civil Engineering Cracow University of Technology, Poland E-mail: plorkisz@cyf-kr.edu.pl, mglowac@gmail.com

KEYWORDS: Evolutionary Algorithms, computation efficiency increase, large non-linear constrained optimization

Development of efficient Evolutionary Algorithms (EA) for a wide class of large non-linear constrained optimization problems is considered. In particular, two important engineering applications are taken into account as target problems, namely residual stress analysis in railroad rails, and vehicle wheels, as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental data. However, the main objective of this research is to develop various algorithmic means of an essential acceleration of the EA based approach for large optimization problems. Moreover, the improved EA should provide possibility of solving such problems, when the standard EA methods fail.

In the analyzed optimization problems, a function given in the discrete form, e.g. expressed in terms of its nodal values, is sought. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. The EA are understood here as real-value coded genetic algorithms consisting of selection, crossover, and mutation operators [1]. Our long-term research includes various ways for increasing efficiency of the EA. So far we have proposed several new acceleration techniques, including solution smoothing and balancing, a'posteriori error analysis and related techniques, an adaptive step-by-step mesh refinement, as well as non-standard parallel and distributed computations [2]. These general ideas can be applied in various ways. For instance, we have proposed two various approaches for smoothing. One of them is based on the moving weighted least squares (MWLS) technique, and the second one uses mean solution curvature. Our a'posteriori solution error analysis is based on the stochastic nature of the EA. In such approach, reference solutions needed for error estimation are generated using weighted averaging of independent solutions taken from different populations, combined with additional smoothing. We have also proposed improved evolutionary operators taking into account information about estimated global and local solution errors.

Moreover, in our research we consider other acceleration techniques, well known in the field of evolutionary computation, including efficient constraint handling techniques, hybrid algorithms combining EA with deterministic methods, as well as parallel and distributed algorithms. So far we have shortly investigated efficient approaches based on penalty functions for constraint handling. We have also studied techniques, recently proposed in literature, for estimation of the convergence point of population.

Results obtained so far are very encouraging and indicate possibilities of further development of speed-up techniques proposed. The efficiency of these techniques was examined using simple but demanding benchmark problems, including residual stress analysis in chosen elastic-perfectly plastic bodies under various cyclic loadings. Chosen PBA benchmark problems, including smoothing of beam deflections obtained by vision measurement system, and reconstruction of residual stresses based on pseudo-experimental data were considered as well. Each of the new speed-up techniques allowed for significant computational efficiency increase. The speed-up factor up to about 140 times was reached so far.

Future research planned includes, inter alia, testing of further new acceleration techniques, as well as application of the improved EA to real large complex engineering problems, including broad PBA data smoothing, and residual stress analysis in railroad rails, and vehicle wheels.

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ISOGEOMETRIC ANALYSIS: IMPORTANT NURBS PROPERTIES

Anna Perduta

Institute for Computational Civil Engineering Cracow University of Technology, Poland E-mail: aperduta@L5.pk.edu.pl

KEYWORDS: isogeometric analysis, NURBS

Isogeometric Analysis (IGA), the recently developed method by Hughes and co-workers [1] has gained strong interest in many research fields. IGA has applications in structural mechanics, fluid mechanics, dynamics, contact mechanics and many others. The method was introduced to fill the gap between CAD modelling and Finite Element Analysis. The basic concept of IGA is to use NURBS (Non-Uniform Rational B-Splines)[2], which are essential tool for modelling geometry in CAD systems, as basis functions in FEM. This allows to transfer geometric model to calculation kernel without additional transformations of geometric model representation. The geometry is described as linear combination of control points treated as degrees of freedom and NURBS functions spanned over knot vector which divides geometry parametric domain into elements. It is worth to emphasize that to represent exact geometry few number of degrees of freedom is necessary as compared with isogeometric representation with Lagrangian shape functions.

As NURBS are the basis for CAD and IGA it is crucial to investigate and understand their properties. The basic properties which make NURBS suitable for several applications are: nonnegativity, partion of unity, linear independance. NURBS curves possess significant property – affine invariance, that is, if one wants to apply transformation to a NURBS curve, it is possible to transform only control points to obtain transformed curve. An important property is the continuity of basis functions, in general a p-th order NURBS is C^{p-1} continuous, this makes IGA suitable for applications in areas where more than C^0 continuity accross the elements' boundary is required, for example in plate/shell formulations, fluid mechanics. Most important properties that characterize NURBS curves and are useful in analysis are variation diminishing property and strong convex hull property. The variation diminishing property is the result of three other NURBS properties: nonnegativity, partition of unity and limited support. Local modification and high continuity of NURBS curves are often used to provide refinement schemes. All properties of NURBS make IGA a powerful tool that in specific fields has many advantages over traditional FEM.

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THE CONSISTENTLY LINEARIZED EIGENPROBLEM AND ITS IMPACT ON AN ENERGY RATIO

S. Pavlicek, H.A. Mang, X. Jia

Institute for Mechanics of Materials and Structures Vienna University of Technology (TU Wien), Austria E-mail: stefan.pavlicek@tuwien.ac.at

KEYWORDS: buckling, consistently linearized eigenproblem, pure membrane stress state, pure bending stress state, finite difference expressions

Helnwein [1] attempted to assess the stability limit of geometrically nonlinear structures *ab initio* by means of the Consistently Linearized Eigenproblem (CLE). The mathematical formulation of the CLE is given as

$$\left[\widetilde{\mathbf{K}}_{T} + (\lambda_{j}^{*} - \lambda)\widetilde{\mathbf{K}}_{T}\right] \cdot \mathbf{v}_{j}^{*} = 0, \qquad j = 1, 2, ..., N,$$
(1)

where $\mathbf{\tilde{K}}_T$ is the tangent stiffness matrix in the frame of the finite element method and $\mathbf{\tilde{K}}_T$ denotes its derivative with respect to the dimensionless load parameter λ . Much later it was realized that a specific eigenvector of the CLE, termed fundamental eigenvector and referred to as \mathbf{v}_1^* , does not depend on λ for the special case of a membrane stress state in the prebuckling regime [2]. This prompted the authors to explore the observed situation on a very much larger scale.

The vertex of the vector \mathbf{v}_1^* describes a curve on a unit sphere. A specific combination of differential geometric quantities related to this curve is found to be equal to the percentage bending energy of the total strain energy. This situation can be visualized by means of a surface curve on an octant of the unit sphere. For convenience's sake it is called *buckling sphere*. The vector **a** (not to be confused with \mathbf{v}_1^*), the vertex of which describes this curve, depends on the zenith angle $\theta(\lambda)$ and the azimuth angle $\phi(\lambda)$. The former is related to the aforementioned percentage bending energy.

Use of the CLE for the analysis of general structures requires commercial FE software. The availability of several types of finite elements and different solvers for large systems of nonlinear equations is a great advantage of such software. The purely numerical output, however, is a disadvantage that affects the ease and the accuracy of subsequent computations, notably of derivatives of scalars, vectors, and matrices.

The hypothesis which links the aforementioned combination of different geometric properties to the percentage bending energy of the total strain energy is verified numerically for the energetical limiting case of a membrane stress state and a pure bending stress state, respectively, in the prebuckling regime. Work concerning verification of the curve described by the vertex of the vector $\mathbf{a}(\theta(\lambda), \phi(\lambda))$ on the buckling sphere for a general stress state is in progress.

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COMPARISON OF RESULTS OF ISOTROPIC AND ANISOTROPIC BEAM MODELS TO RESULTS OBTAINED BY 3D FINITE ELEMENT SIMULATIONS

Máté Hazay

Department of Structural Mechanics Budapest University of Technology and Economics, Hungary E-mail: matehazay@hotmail.com

KEYWORDS: classical beam theory, Timoshenko beam theory, effect of shear deformations, higher-order beam theories, analytical solutions, sandwich beams, anisotropic elasticity, finite element simulations

Analysis of the mechanical behaviour of beams lasts for centuries, and several researches led to the evolution of many different beam theories. Each model contains certain assumptions and negligence comparing to the classical continuum mechanical approach, which cause that their results differ from the exact solution. Considering the fact that these differences in most cases are relatively small, beam theories are widespread and widely used among engineers. However, in order to perform reliable calculations, engineers should be aware of the applicability limits of the approximate method they are working with.

In this study several beam theories were used to determine the stresses and deflections of beams made of homogeneous and isotropic material. Furthermore, sandwich beams and beams made of anisotropic material were also analyzed. The applied theories were evaluated by performing a comparative analysis, where their results were compared to "accurate" results obtained by 3D finite element simulations. The main goal of this study was to determine the magnitude of the error beam theories working with, and to make a recommendation about the applicability of these methods. The results of the beam theories were achieved by using analytical solutions which were obtained for simple cantilever structures. When the analyzed beam was made of anisotropic material, the solution was obtained by using the theory of anisotropic elasticity based on [1] and [2].

Results show that in case of compact isotropic beams the solution of beam theories are quite reliable except for very short and thick beams. However, since the effect of the shear deformations highly depends on the shape of the cross section, in case of beams having thin-walled sections the results may differ significantly from the exact solution. In case of sandwich beams the effect of shear deformations are even larger, which may lead to significant errors in the results. Furthermore, the reliability of the results are highly depend on the geometrical and the stiffness characteristics of the sandwich cross section, therefore sometimes strict applicability limits should be assigned. During the analysis of anisotropic beams the relatively complicated solutions based on anisotropic elasticity led to relatively accurate results, although the applied closed-form solutions are available only for relatively simple problems.

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Notes

ISBN: 978-83-7242-684-0