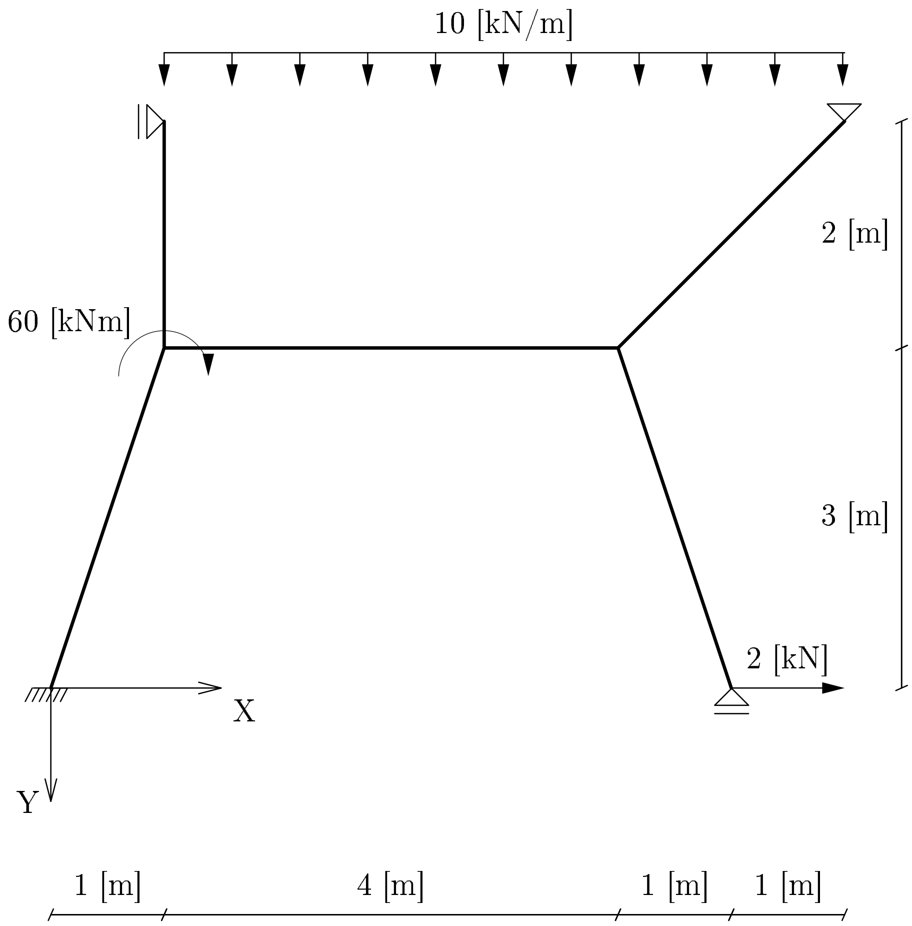
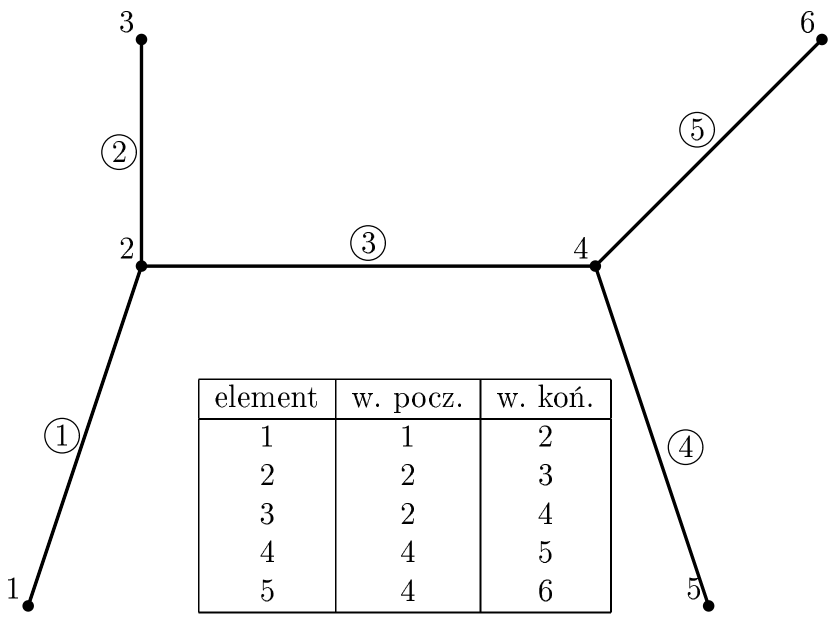
Jan Jaśkowiec 13 kwiecien 2003

Analiza statyczna ramy metodą elementów skończonych



$A = 2.3 \cdot 10^{-2}\,\mathrm{m}^2$
$J = 1.0 \cdot 10^{-4}\,\mathrm{m}^4$
$E = 210$ [GPa]



element	w. pocz.	w. koń.
1	1	2
2	2	3
3	2	4
4	4	5
5	4	6

$E := 10 \cdot 10^{6}$ $\qquad$ $ORIGIN := 1$

$A := 2.3 \cdot 10^{-2}$

$I := 1 \cdot 10^{-4}$

$EA := E \cdot A$ $\qquad$ $EI := E \cdot I$

$$ke(L) := \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ 0 & 12 \cdot \frac{EI}{L^3} & 6 \cdot \frac{EI}{L^2} & 0 & -12 \cdot \frac{EI}{L^3} & 6 \cdot \frac{EI}{L^2} \\ 0 & 6 \cdot \frac{EI}{L^2} & 4 \frac{EI}{L} & 0 & -6 \cdot \frac{EI}{L^2} & 2 \cdot \frac{EI}{L} \\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -12 \cdot \frac{EI}{L^3} & -6 \cdot \frac{EI}{L^2} & 0 & 12 \cdot \frac{EI}{L^3} & -6 \cdot \frac{EI}{L^2} \\ 0 & 6 \cdot \frac{EI}{L^2} & 2 \cdot \frac{EI}{L} & 0 & -6 \cdot \frac{EI}{L^2} & 4 \cdot \frac{EI}{L} \end{bmatrix} \Gamma e(a) := \begin{bmatrix} \cos(a) & \sin(a) & 0 & 0 & 0 & 0 \\ -\sin(a) & \cos(a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(a) & \sin(a) & 0 \\ 0 & 0 & 0 & -\sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$L1 := \sqrt{10}$ $\qquad$ $L2 := 2$ $\qquad$ $L3 := 4$ $\qquad$ $L4 := \sqrt{10}$ $\qquad$ $L5 := \sqrt{8}$

$a1 := -\operatorname{acos}\left(\frac{2}{L1}\right)$ $\qquad$ $a2 := \frac{-\pi}{2}$ $\qquad$ $a3 := 0$ $\qquad$ $a4 := \operatorname{acos}\left(\frac{1}{L4}\right)$ $\qquad$ $a5 := -\operatorname{acos}\left(\frac{2}{L5}\right)$

Macierze agregacji

$B1_{6,18} := 0$ $\qquad$ $B2_{6,18} := 0$ $\qquad$ $B3_{6,18} := 0$ $\qquad$ $B4_{6,18} := 0$ $\qquad$ $B5_{6,18} := 0$

$B1_{1,1} := 1$ $\qquad$ $B1_{2,2} := 1$ $\qquad$ $B1_{3,3} := 1$ $\qquad$ $B1_{4,4} := 1$ $\qquad$ $B1_{5,5} := 1$ $\qquad$ $B1_{6,6} := 1$

$B2_{1,4} := 1$ $\qquad$ $B2_{2,5} := 1$ $\qquad$ $B2_{3,6} := 1$ $\qquad$ $B2_{4,7} := 1$ $\qquad$ $B2_{5,8} := 1$ $\qquad$ $B2_{6,9} := 1$

$B3_{1,4} := 1$ $\qquad$ $B3_{2,5} := 1$ $\qquad$ $B3_{3,6} := 1$ $\qquad$ $B3_{4,10} := 1$ $\qquad$ $B3_{5,11} := 1$ $\qquad$ $B3_{6,12} := 1$

$B4_{1,10} := 1$ $\qquad$ $B4_{2,11} := 1$ $\qquad$ $B4_{3,12} := 1$ $\qquad$ $B4_{4,13} := 1$ $\qquad$ $B4_{5,14} := 1$ $\qquad$ $B4_{6,15} := 1$

$B5_{1,10} := 1$ $\qquad$ $B5_{2,11} := 1$ $\qquad$ $B5_{3,12} := 1$ $\qquad$ $B5_{4,16} := 1$ $\qquad$ $B5_{5,17} := 1$ $\qquad$ $B5_{6,18} := 1$

Macierze sztywnosci

$k1 := ke(L1)$ $k2 := ke(L2)$ $k3 := ke(L3)$ $k4 := ke(L4)$ $k5 := ke(L5)$

$T1 := Te(a1)$ $T2 := Te(a2)$ $T3 := Te(a3)$ $T4 := Te(a4)$ $T5 := Te(a5)$

$K1 := T1^T \cdot k1 \cdot T1$ $K2 := T2^T \cdot k2 \cdot T2$ $K3 := T3^T \cdot k3 \cdot T3$ $K4 := T4^T \cdot k4 \cdot T4$ $K5 := T5^T \cdot k5 \cdot T5$

Agregacja do globalnej macierzy sztywnoœci

$$K := B1^T \cdot K1 \cdot B1 + B2^T \cdot K2 \cdot B2 + B3^T \cdot K3 \cdot B3 + B4^T \cdot K4 \cdot B4 + B5^T \cdot K5 \cdot B5$$

Macierz funkcji ksztaltu

$$Nk1(x, L) := 1 - \frac{x}{L} \qquad Nk2(x, L) := \frac{x}{L}$$

$$Nb1(x, L) := 1 - 3 \cdot \left(\frac{x}{L}\right)^2 + 2 \cdot \left(\frac{x}{L}\right)^3 \qquad Nb2(x, L) := L \cdot \left[\left(\frac{x}{L}\right) - 2 \cdot \left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3\right]$$

$$Nb3(x, L) := 3 \cdot \left(\frac{x}{L}\right)^2 - 2 \cdot \left(\frac{x}{L}\right)^3 \qquad Nb4(x, L) := L \cdot \left[\left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\right]$$

$$N(x, L) := \begin{bmatrix} Nk1(x, L) & 0 & 0 & Nk2(x, L) & 0 & 0 \\ 0 & Nb1(x, L) & Nb2(x, L) & 0 & Nb3(x, L) & Nb4(x, L) \end{bmatrix}$$

Wektor prawej strony

$q3(x) := \begin{bmatrix} 0 \\ 10 \end{bmatrix}$ $q5(x) := \begin{bmatrix} 0 \\ 10 \end{bmatrix}$

$L3c := 4$ $L5c := 2$

$i := 1 .. 6$

$$P3_i := \int_0^{L3c} N(x, L3c)_{1,i} \cdot q3(x)_1 + N(x, L3c)_{2,i} \cdot q3(x)_2 \, dx$$

$$P5_i := \int_0^{L5c} N(x, L5c)_{1,i} \cdot q3(x)_1 + N(x, L5c)_{2,i} \cdot q3(x)_2 \, dx$$

$Pw_{18} := 0$

$Pw_6 := 60$ $Pw_{13} := 2$

$F := B3^T \cdot P3 + B5^T \cdot P5 + Pw$

Wektor warunków brzegowych

$$Wb := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 7 \\ 14 \\ 16 \\ 17 \end{bmatrix}$$

$Kb := K$ $Fb := F$

Warunki brzegowe

$i := 1 .. 18$
$j := 1 .. 7$

$Kb_{i, Wb_j} := 0$ $Kb_{(Wb_j), i} := 0$

$Kb_{Wb_j, Wb_j} := 1$

$Fb_{(Wb_j)} := 0$

Wektor rozwiązania

$Q := Kb^{-1} \cdot Fb$

Wektor reakcji

$R := K \cdot Q - F$

reakcje dla wêz³a 1

$R_1 = 9.819$ $R_2 = -22.777$ $R_3 = -18.434$

reakcje dla wêz³a 2

$R_4 = 1.119 \cdot 10^{-13}$ $R_5 = -2.522 \cdot 10^{-13}$ $R_6 = 0$

reakcje dla wêz³a 3

$R_7 = -34.508$ $R_8 = 3.039 \cdot 10^{-13}$ $R_9 = 1.041 \cdot 10^{-14}$

reakcje dla wêz³a 4

$R_{10} = 4.494 \cdot 10^{-13}$ $R_{11} = 2.736 \cdot 10^{-13}$ $R_{12} = 2.842 \cdot 10^{-14}$

reakcje dla wêz³a 5

$R_{13} = -9.259 \cdot 10^{-14}$ $R_{14} = -9.907$ $R_{15} = -8.736 \cdot 10^{-15}$

reakcje dla wêz³a 6

$R_{16} = 22.688$ $R_{17} = -27.316$ $R_{18} = 5.773 \cdot 10^{-15}$

Równowaga globalna

Równowaga na osi X

$R_1 + R_7 + R_{16} + 2 = -1.563 \cdot 10^{-13}$

Równowaga na osi Y

$R_2 + R_{14} + R_{17} + 60 = 1.776 \cdot 10^{-13}$

Równowaga momentu względem punktu A(0,0)

$R_3 + R_7 \cdot 5 + R_{14} \cdot 6 + R_{16} \cdot 5 + R_{17} \cdot 6 + 10 \cdot 6 \cdot 3 + 60 = -60.867$

Siły przywezłowe

element 1

$Q1 := B1 \cdot Q$

$S1 := K1 \cdot Q1$

$s1 := T1^{T} \cdot S1$

$$s1 = \begin{bmatrix} -11.433 \\ -22.012 \\ -18.434 \\ 11.433 \\ 22.012 \\ -3.068 \end{bmatrix}$$

element 2

$Q2 := B2 \cdot Q$

$S2 := K2 \cdot Q2$

$s2 := T2^{T} \cdot S2$

$$s2 = \begin{bmatrix} -3.018 \cdot 10^{-13} \\ -34.508 \\ 69.015 \\ 3.018 \cdot 10^{-13} \\ 34.508 \\ 1.041 \cdot 10^{-14} \end{bmatrix}$$

element 3

$Q3 := B3 \cdot Q$

$S3 := K3 \cdot Q3 - P3$

$s3 := T3^{T} \cdot S3$

$$s3 = \begin{bmatrix} -24.688 \\ -22.777 \\ -5.947 \\ 24.688 \\ -17.223 \\ -5.162 \end{bmatrix}$$

element 4

$Q4 := B4 \cdot Q$

$S4 := K4 \cdot Q4$

$s4 := T4^{T} \cdot S4$

$$s4 = \begin{bmatrix} -10.031 \\ 1.235 \\ 15.907 \\ 10.031 \\ -1.235 \\ -8.736 \cdot 10^{-15} \end{bmatrix}$$

element 5

$Q5 := B5 \cdot Q$

$S5 := K5 \cdot Q5 - P5$

$s5 := T5^{T} \cdot S5$

$$s5 = \begin{bmatrix} -10.87 \\ 21.216 \\ -10.745 \\ -3.272 \\ -35.358 \\ 5.773 \cdot 10^{-15} \end{bmatrix}$$