Computational methods
- modelling and simulation

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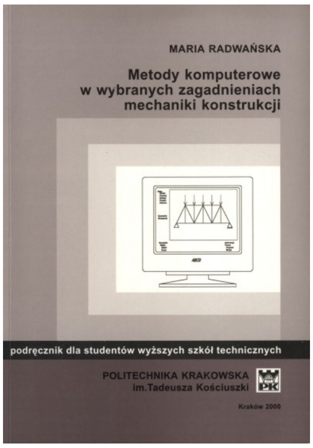
Lecture contents

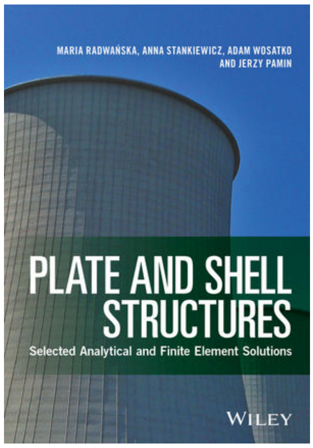
- Scope and reading
- Modelling of physical phenomena
- Finite Element Method *(Metoda Elementów Skończonych)*
- Computer simulations in civil engng and mathematical physics

With thanks to:
Authors of presented simulations
C.A. Felippa (Univ. of Colorado at Boulder)
www.colorado.edu/engineering/cas/courses.d/IFEM.d
TNO DIANA http://www.tnodiana.com
ADINA R&D, Inc. http://www.adina.com
ANSYS, Inc. http://www.ansys.com
R.D. Cook, *Finite Element Method for Stress Analysis*, Wiley 1995
T. Kolendowicz *Mechanika budowli dla architektów*, Arkady 1996
Co-workers from Institute L-5, CED, CUT

Course scope and reading

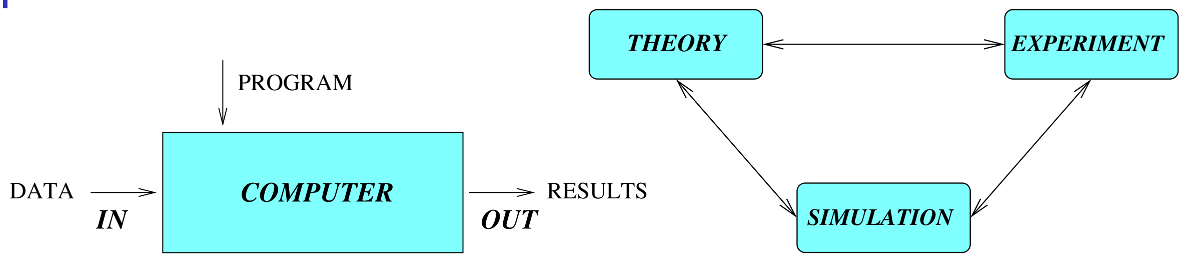
- Lectures (theoretical background, examples)
- Laboratory classes (ROBOT, MATLAB, CALFEM, ABAQUS)
- Exercise and 4 assignments
- Tests to pass 2 parts of the course





Plenty of books in English:
Cook, Felippa, Ottosen & Petersson, Zienkiewicz & Taylor

Computational methods



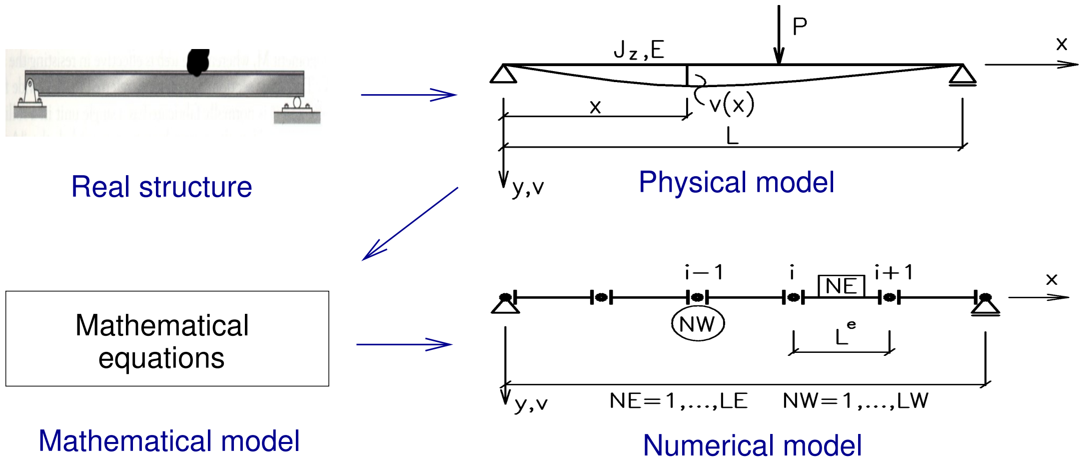
- Computational method is the manner of analysis of a problem using approximate calculation algorithms implemented as computer programs.
- Owing to robustness of modern approximation methods and advanced number-crunching capabilities of computers it is possible to search for an optimal solution using computer simulations.

Computer simulations

- substitute/assist experimental research (on real models of original objects)
- substitute/assist analytical methods (but do not substitute modelling process)

Modelling process

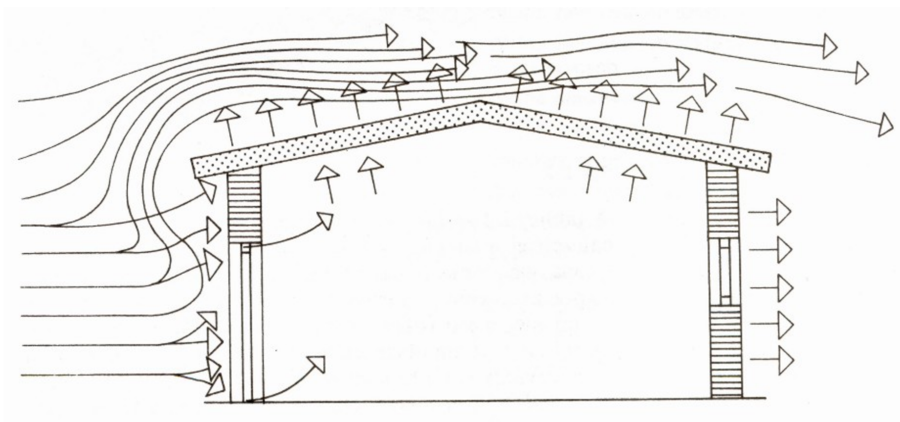
Model sequence in structural mechanics



Aim: to obtain a simple model, accounting for the most important structure properties and its response to applied loading, and analyzable by computation tools.

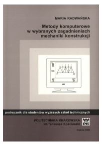
Modelling process

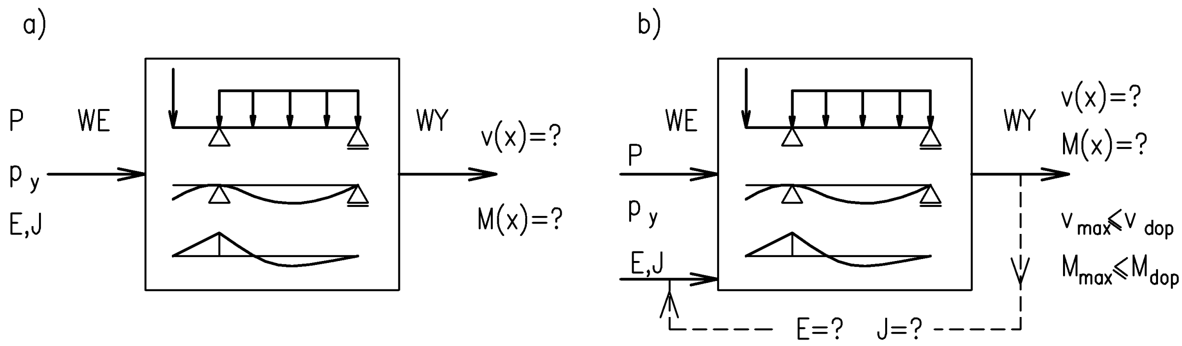
Idealization, simplification, approximation



Set of assumptions: model of structure, material, loading
Physical model: representation of essential features
Mathematical model: set of equations (algebraic, differential, integral) + boundary/initial conditions

Analysis and synthesis of structures





Analysis Synthesis (design)

Physical models

Dimensionality reduction:

- bar (beam, frame) structures - one-dimensional
- panels, plates and shells - two-dimensional
- three-dimensional

Physical and mathematical models

Changes in time:

- stationary problems - independent of time (statics)
- nonstationary problems - dependent on time (evolutionary, dynamics)

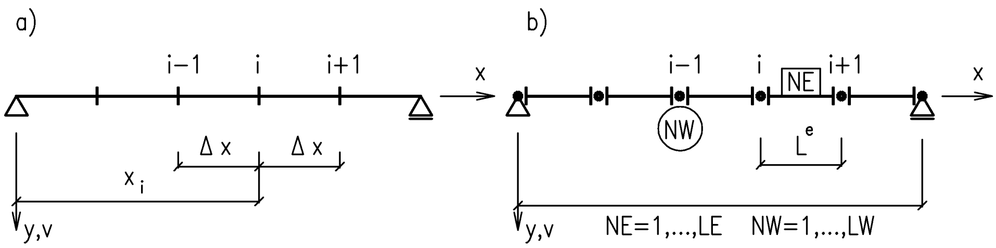
Simplification based on hypotheses:

- kinematic (geometrical), e.g. dominating dimensions, cross-section type
- static/dynamic - e.g. loads changing fast or slowly, loads action in one plane

Mathematical models are:

- linear (small deformations and Hooke's law) $\longrightarrow$ superposition principle holds
- nonlinear

Computational model



Analytical solution of a continuous problem or

numerical solution for a discrete problem

Discretization of a problem

Finite Difference Method - FDM (*MRS*)

Finite Element Method - FEM (*MES*)

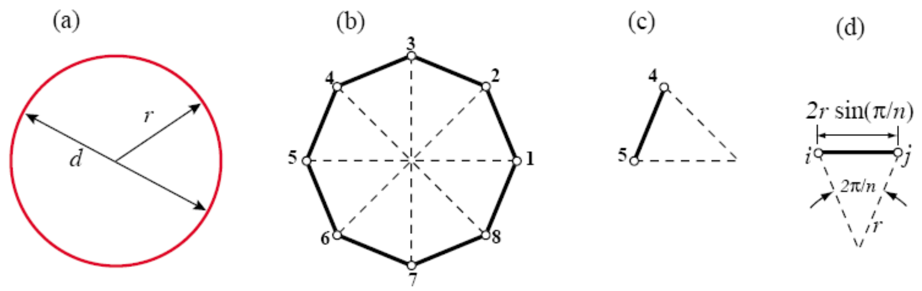
Boundary Element Method - BEM (*MEB*)

Why is it worthwhile to learn FEM?

- For many practical engineering problems it is impossible to find an analytical solution (complex domain, loading or nonlinearities)
- Owing to FEM simulations one can easily understand the behaviour of a system and cheaply analyze the influence of various model parameters on the approximate solution
- It is possible to take into account more important aspects than if the solution were analytical
- Knowledge of FEM is necessary for modern engineer since it is a universal and dominating computational technology

Remark:
Without understanding the physics and theoretical foundations of FEM it is possible to obtain results, but virtually impossible to evaluate the results

FEM idea



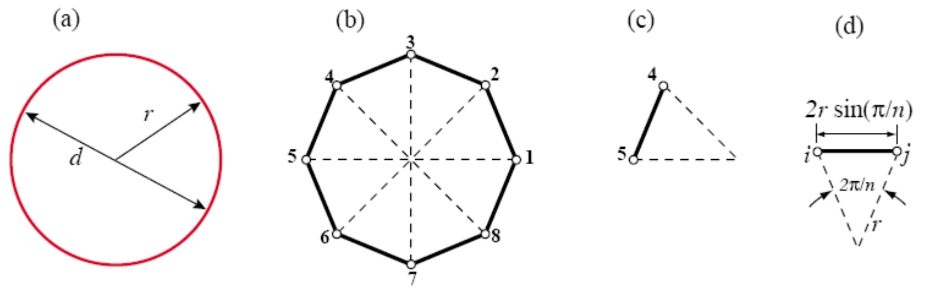
Assignment: find circumference L of a circle with diameter $d = 2r$.

Exact solution: $L = \pi d$.

Discrete solution: build a polygon with n sides, determine each edge length L_{ij}, compute polygon circumference $\tilde{L} = nL_{ij}$, increase n to obtain a more accurate approximation of the circle circumference until $\tilde{L} \approx L$

If $d = 1$, $n = 4 \longrightarrow \tilde{L} \approx 2.8284$, $n = 32 \longrightarrow \tilde{L} \approx 3.1365$.

FEM idea

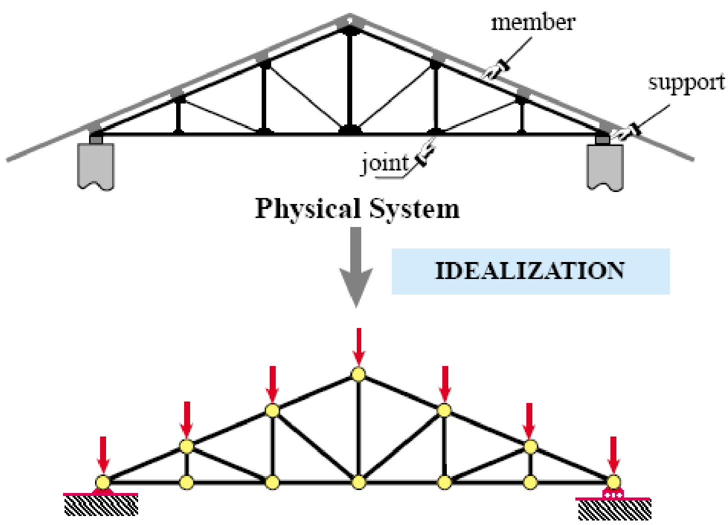


Discrete approximation:

edges ⟶ finite elements (*elementy skończone*)
vertices ⟶ nodes (*węzły*)
decomposition into elements ⟶ disassembly (*siatka MES*)
computation of L_{ij} ⟶ analysis of generic element (*interpolacja*)
connection of n elements ⟶ assembly (*agregacja*)
computation of circumference $\tilde{L}$ ⟶ solution (*rozwiązanie*)

The idea of FEM comes form Egiptian mathematicians (1800 BC) and Archimedes (250 BC), but the fast progress of the method was enabled by computers (since the sixties).

Physical interpretation of FEM



Simplified manner to obtain a discrete FEM model of a structure
The behaviour of an element is characterized by its nodal degrees of freedom (dofs)
The response of a structure is determined by the elements and their interactions

Numerical model

Set of linear equations

$$\mathbf{Kd} = \mathbf{f}$$

K - stiffness matrix

d - vector of dofs

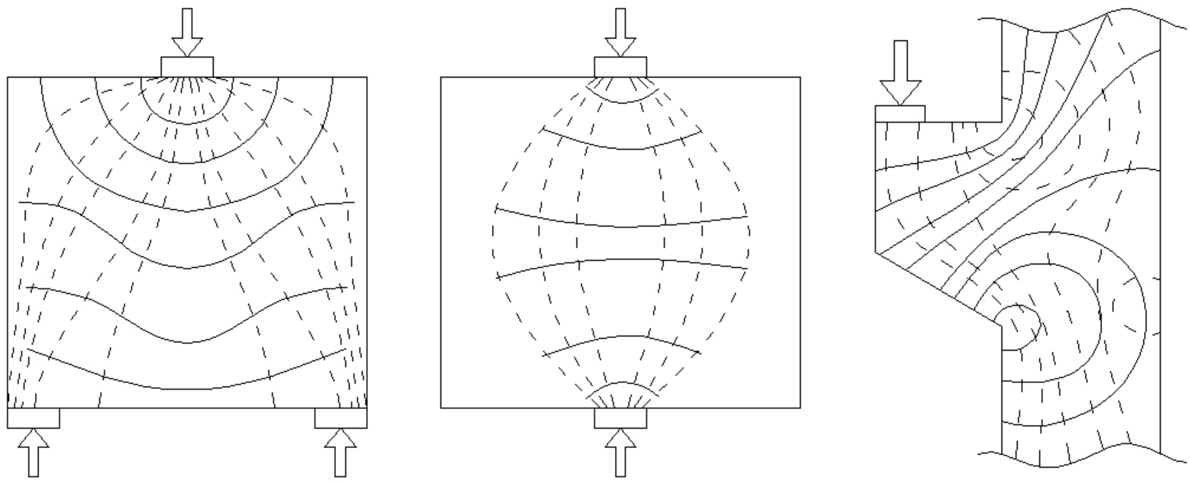
f - load vector

Similar model for various stationary problems of physics

Errors in FEM modelling

- Modelling error
- Discretization error
- Solution error

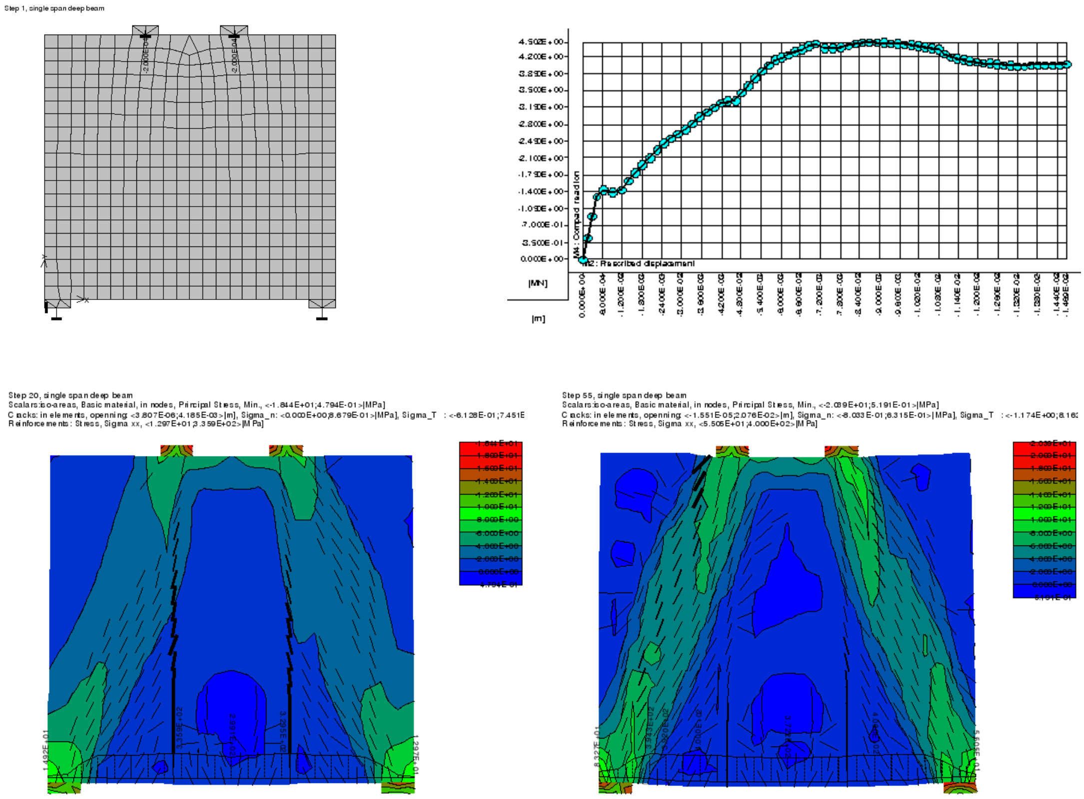
Understanding how a structure responds



______________tension

- - - - - - - - - - - compression

Nonlinear analysis of reinforced concrete panel using ATENA package (M. Kwasek)



Simulations in civil engineering and physics

TNO DIANA `http://www.tnodiana.com`
Four-span slab under moving load
Building under earthquake load
Air flow around a chimney

ADINA R&D, Inc.`http://www.adina.com`
Helmet Car Dam

ANSYS, Inc. `http://www.ansys.com`
Shell Seal Donut