# Computational methods - modelling and simulation

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### Lecture contents

- Scope and reading
- Modelling of physical phenomena
- Finite Element Method (Metoda Elementów Skończonych)
- Computer simulations in civil engng and mathematical physics

#### With thanks to:

AUTHORS OF PRESENTED SIMULATIONS C.A. FELIPPA (UNIV. OF COLORADO AT BOULDER) www.colorado.edu/engineering/cas/courses.d/IFEM.d TNO DIANA http://www.tnodiana.com ADINA R&D, INC. http://www.adina.com ANSYS, INC. http://www.ansys.com R.D. COOK, *Finite Element Method for Stress Analysis*, WILEY 1995 T. KOLENDOWICZ *Mechanika budowli dla architektów*, ARKADY 1996 CO-WORKERS FROM INSTITUTE L-5, CED, CUT



## Course scope and reading

- Lectures (theoretical background, examples)
- Laboratory classes (ROBOT, ABAQUS)
- Exercise and 4 assignments
- Tests to pass 2 parts of the course



Plenty of books in English: Cook, Felippa, Ottosen & Petersson, Zienkiewicz & Taylor



## Computational methods



- Computational method is the manner of analysis of a problem using approximate calculation algorithms implemented as computer programs.
- Owing to robustness of modern approximation methods and advanced number-crunching capabilities of computers it is possible to search for an optimal solution using computer simulations.

#### Computer simulations

- substitute/assist experimental research (on real models of original objects)
- substitute/assist analytical methods (but do not substitute modelling process)



## Modelling process

### Model sequence in structural mechanics



Aim: to obtain a simple model, accounting for the most important structure properties and its response to applied loading, and analyzable by computation tools.

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## Modelling process

Idealization, simplification, approximation



Set of assumptions: model of structure, material, loading Physical model: representation of essential features Mathematical model: set of equations (algebraic, differential, integral) + boundary/initial conditions



# Analysis and synthesis of structures



Analysis

Synthesis (design)

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## Physical models



#### Dimensionality reduction:

- bar (beam, frame) structures one-dimensional
- panels, plates and shells two-dimensional
- three-dimensional



## Physical and mathematical models

### Changes in time:

- stationary problems independent of time (statics)
- nonstationary problems dependent on time (evolutionary, dynamics)

### Simplification based on hypotheses:

- kinematic (geometrical), e.g. dominating dimensions, cross-section type
- static/dynamic e.g. loads changing fast or slowly, loads action in one plane

### Mathematical models are:

- ► linear (small deformations and Hooke's law) → superposition principle holds
- nonlinear

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KTIwł

## Computational model



Analytical solution of a continuous problem or numerical solution for a discrete problem Discretization of a problem

Finite Difference Method - FDM (*MRS*) Finite Element Method - FEM (*MES*) Boundary Element Method - BEM (*MEB*)

### Why is it worthwhile to learn FEM?

- For many practical engineering problems it is impossible to find an analytical solution (complex domain, loading or nonlinearities)
- Owing to FEM simulations one can easily understand the behaviour of a system and cheaply analyze the influence of various model parameters on the approximate solution
- It is possible to take into account more important aspects than if the solution were analytical
- Knowledge of FEM is necessary for modern engineer since it is a universal and dominating computational technology

#### Remark:

Without understanding the physics and theoretical foundations of FEM it is possible to obtain results, but virtually impossible to evaluate the results

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## FEM idea



Assignment: find circumference *L* of a circle with diameter d = 2r. Exact solution:  $L = \pi d$ .

Discrete solution: build a polygon with *n* sides, determine each edge length  $L_{ij}$ , compute polygon circumference  $\tilde{L} = nL_{ij}$ , increase *n* to obtain a more accurate approximation of the circle circumference until  $\tilde{L} \approx L$ 

If d = 1,  $n = 4 \longrightarrow \tilde{L} \approx 2.8284$ ,  $n = 32 \longrightarrow \tilde{L} \approx 3.1365$ .

# FEM idea



### Discrete approximation:

edges  $\rightarrow$  finite elements (*elementy skończone*) vertices  $\rightarrow$  nodes (*węzły*) decomposition into elements  $\rightarrow$  disassembly (*siatka MES*) computation of  $L_{ij} \rightarrow$  analysis of generic element (*interpolacja*) connection of *n* elements  $\rightarrow$  assembly (*agregacja*) computation of circumference  $\tilde{L} \rightarrow$  solution (*rozwiązanie*)

The idea of FEM comes form Egiptian mathematicians (1800 BC) and Archimedes (250 BC), but the fast progress of the method was enabled by computers (since the sixties).

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# Physical interpretation of FEM



Simplified manner to obtain a discrete FEM model of a structure The behaviour of an element is characterized by its nodal degrees of freedom (dofs)

The response of a structure is determined by the elements and their interactions



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## Numerical model

Set of linear equations

# $\mathbf{K}\mathbf{d} = \mathbf{f}$

- K stiffness matrix
- **d** vector of dofs
- f load vector

Similar model for various stationary problems of physics

### Errors in FEM modelling

- Modelling error
- Discretization error
- Solution error

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## Understanding how a structure responds



\_ \_ \_ \_ \_ \_ compression



# Nonlinear analysis of reinforced concrete panel using ATENA package (M. Kwasek)





# Simulations in civil engineering and physics

TNO DIANA http://www.tnodiana.com Four-span slab under moving load Building under earthquake load Air flow around a chimney

ADINA R&D, Inc.http://www.adina.com Helmet Car Dam

ANSYS, Inc. http://www.ansys.com Shell Seal Donut



