FEM solution for example problem of stationary heat flow

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perfunction
$$N_i(x, y) = \alpha_{1i} + \alpha_{2i}x^e + \alpha_{3i}y$$

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\Delta}$$

$$W_{\alpha_{1i}} = \begin{vmatrix} 1 & x_i & y_i \\ 0 & x_j & y_j \\ 0 & x_k & y_k \end{vmatrix} = x_j y_k - x_k y_j$$





perfunction
$$N_i(x, y) = \alpha_{1i} + \alpha_{2i}x^e + \alpha_{3i}y$$

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
 $W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\Delta}$
 $W_{\alpha_{1i}} = \begin{vmatrix} 1 & x_i & y_i \\ 0 & x_j & y_j \\ 0 & x_k & y_k \end{vmatrix} = x_j y_k - x_k y_j$
 $\alpha_{1i} = \frac{W_{\alpha_{1i}}}{W} = \frac{x_j y_k - x_k y_j}{2P_{\Delta}}$





Shape function
$$N_i(x, y) = \alpha_{1i} + \alpha_{2i}x^e + \alpha_{3i}y$$

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\triangle}$$

$$W_{\alpha_{2i}} = \begin{vmatrix} 1 & 1 & y_i \\ 1 & 0 & y_j \\ 1 & 0 & y_k \end{vmatrix} = y_j - y_k$$





De function
$$N_i(x, y) = \alpha_{1i} + \alpha_{2i}x^e + \alpha_{3i}y$$

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\Delta}$$

$$W_{\alpha_{2i}} = \begin{vmatrix} 1 & 1 & y_i \\ 1 & 0 & y_j \\ 1 & 0 & y_k \end{vmatrix} = y_j - y_k$$

$$\alpha_{2i} = \frac{W_{\alpha_{2i}}}{W} = \frac{y_j - y_k}{2P_{\Delta}}$$







$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\Delta}$$
$$W_{\alpha_{3i}} = \begin{vmatrix} 1 & x_i & 1 \\ 1 & x_j & 0 \\ 1 & x_k & 0 \end{vmatrix} = x_k - x_j$$





be function
$$N_i(x, y) = \alpha_{1i} + \alpha_{2i}x^e + \alpha_{3i}y$$

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$W = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 2P_{\Delta}$$

$$W_{\alpha_{3i}} = \begin{vmatrix} 1 & x_i & 1 \\ 1 & x_j & 0 \\ 1 & x_k & 0 \end{vmatrix} = x_k - x_j$$

$$\alpha_{3i} = \frac{W_{\alpha_{3i}}}{W} = \frac{x_k - x_j}{2P_{\Delta}}$$













$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma_{\mathrm{q}}} w h \widehat{q} \mathrm{d} \Gamma - \int_{\Gamma_{\mathrm{T}}} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

+ boundary condition

$$T=\widehat{T}$$
 on Γ_{T}



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\mathbf{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\mathbf{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\boldsymbol{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\boldsymbol{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \text{const}$$
$$\mathbf{w}^{\mathrm{T}}\int_{A}\mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \boldsymbol{\Theta} = -\mathbf{w}^{\mathrm{T}}\int_{\Gamma}\mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \mathbf{w}^{\mathrm{T}}\int_{A}\mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \forall \mathbf{w}$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\mathbf{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\mathbf{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \text{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \mathbf{\Theta} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\Theta, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\Theta$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \Theta = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A$$
$$\mathbf{K} = \int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A, \quad \mathbf{f} = \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \mathbf{f}_{b} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

+ boundary condition

$$T = \widehat{T}$$
 on Γ_{T}

$$T = \mathbf{N}\Theta, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\Theta$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \Theta = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A$$
$$\mathbf{K} = \int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A, \quad \mathbf{f} = \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \mathbf{f}_{b} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma$$
$$\mathbf{K}\Theta = \mathbf{f} + \mathbf{f}_{b}$$



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 ${\bf K}$ matrix – element 1

$$\mathbf{N}^1 = \begin{bmatrix} 1 - \frac{1}{4}x & \frac{1}{4}x - \frac{1}{3}y & \frac{1}{3}y \end{bmatrix}$$







${\bf K}$ matrix – element 1

$$\mathbf{N}^{1} = \begin{bmatrix} 1 - \frac{1}{4}x & \frac{1}{4}x - \frac{1}{3}y & \frac{1}{3}y \end{bmatrix}$$
$$\mathbf{B}^{1} = \nabla \mathbf{N} = \begin{bmatrix} -0.250 & 0.250 & 0.000\\ 0.000 & -0.333 & 0.333 \end{bmatrix}$$





${\bf K}$ matrix – element 1

$$\mathbf{N}^{1} = \begin{bmatrix} 1 - \frac{1}{4}x & \frac{1}{4}x - \frac{1}{3}y & \frac{1}{3}y \end{bmatrix}$$
$$\mathbf{B}^{1} = \nabla \mathbf{N} = \begin{bmatrix} -0.250 & 0.250 & 0.000 \\ 0.000 & -0.333 & 0.333 \end{bmatrix}$$
$$\mathbf{K}^{1} = \int_{A^{1}} \mathbf{B}^{\mathrm{T}} k \mathbf{B} \mathrm{d} A = A^{1} \mathbf{B}^{\mathrm{T}} k \mathbf{B}$$
$$= \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$





${f K}$ matrix – element 2

$$\mathbf{N}^2 = \begin{bmatrix} 1 - \frac{1}{3}y & \frac{1}{4}x & \frac{1}{3}y - \frac{1}{4}x \end{bmatrix}$$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$





\mathbf{K} matrix – element 2

 $q_n = 5 \text{ J/m}^2 \text{s}$

 $k = 0.9 \text{ J/ms}^{\circ}\text{C}$ $f = 2 \text{ J/m}^{2}\text{s}$ h = 1 m

 $q_n = 0$

 $= 20^{\circ} C$

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$$\mathbf{N}^{2} = \begin{bmatrix} 1 - \frac{1}{3}y & \frac{1}{4}x & \frac{1}{3}y - \frac{1}{4}x \end{bmatrix}$$
$$\mathbf{B}^{2} = \nabla \mathbf{N} = \begin{bmatrix} 0.000 & 0.250 & -0.250 \\ -0.333 & 0.000 & 0.333 \end{bmatrix}$$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$



${\bf K}$ matrix – element 2

$$\mathbf{N}^{2} = \begin{bmatrix} 1 - \frac{1}{3}y & \frac{1}{4}x & \frac{1}{3}y - \frac{1}{4}x \end{bmatrix}$$
$$\mathbf{B}^{2} = \nabla \mathbf{N} = \begin{bmatrix} 0.000 & 0.250 & -0.250 \\ -0.333 & 0.000 & 0.333 \end{bmatrix}$$
$$\mathbf{K}^{2} = \int_{A^{2}} \mathbf{B}^{\mathrm{T}} k \mathbf{B} \mathrm{d} A = A^{2} \mathbf{B}^{\mathrm{T}} k \mathbf{B}$$
$$= \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix}$$



$$\mathbf{K}^1 = \left[\begin{array}{ccc} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{array} \right.$$



${f f}$ vector – element 1 and 2 - $A^1=A^2$

$$\mathbf{f}^e = \int_{A^e} \mathbf{N}^{\mathrm{T}} f \mathrm{d}A = \frac{f}{3} A^e \begin{bmatrix} 1\\1\\1 \end{bmatrix} = \begin{bmatrix} 4\\4\\4 \end{bmatrix}$$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$
$$\mathbf{K}^{2} = \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix}$$



 $= 20^{\circ} C$

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\mathbf{f}_b vector – element 1

$$\begin{split} \mathbf{f}_{b}^{1} &= -\int_{\Gamma_{ij}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \\ &- \int_{\Gamma_{ki}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \end{split}$$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$
$$\mathbf{K}^{2} = \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix}$$
$$\mathbf{f}^{1} = \mathbf{f}^{2} = \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix}$$



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$$\mathbf{f}_{b}^{1} = -\int_{\Gamma_{ij}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathbf{n}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathbf{n}} \mathrm{d}\Gamma$$
flow continuity
along edge 1-3
$$q_{nki}^{1} = -q_{nij}^{2}$$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$
$$\mathbf{K}^{2} = \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix}$$
$$\mathbf{f}^{1} = \mathbf{f}^{2} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$





\mathbf{f}_b vector – element 1

h = 1 m

 $q_n = 0$

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$$\mathbf{f}_{b}^{1} = -\int_{\Gamma_{ij}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} \mathbf{q}_{\mathbf{n}} \, \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} \mathbf{q}_{\mathbf{n}} \, \mathrm{d}\Gamma$$
flow continuity
along edge 1-3
$$q_{nki}^{1} = -q_{nij}^{2}$$

$$\mathbf{f}_{b}^{1} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$$

$$\mathbf{q}_{nki}^{q_{nki}} = -q_{nij}^{2}$$

$$\mathbf{f}_{b}^{1} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$$

$$\mathbf{f}_{b}^{1} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \end{split}$$



$$\begin{split} \mathbf{f}_{b}^{2} &= -\int_{\Gamma_{ij}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \\ &- \int_{\Gamma_{jk}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{ki}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \end{split}$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}, \end{split}$$





$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \end{split}$$

flow continuity

$$\mathbf{f}_{b}^{2} = -\int_{\Gamma_{ij}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \not\stackrel{\mathbf{f}}{=} \frac{\mathrm{along edge 1-3}}{q_{n_{ki}}^{1} = -q_{n_{ij}}^{2}} \\
-\int_{\Gamma_{jk}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{ki}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$





$$\mathbf{f}_{b}^{2} = -\int_{\Gamma_{jk}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{ki}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

$$-\int_{\Gamma_{jk}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma = -\int_0^4 \left(\mathbf{N}^2(x, y=3) \right)^{\mathrm{T}} (-5) \mathrm{d}x$$
$$= \begin{bmatrix} 0\\10\\10 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} \\ \mathbf{f}_{b}^{1} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \end{split}$$



\mathbf{f}_b vector – elem<u>ent 2</u>

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 $= 20^{\circ}$ C k =h =

$$\mathbf{f}_b^2 = \begin{bmatrix} 0\\10\\10 \end{bmatrix} - \int_{\Gamma_{ki}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

$$-\int_{\Gamma_{ki}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma = -\int_0^3 \left(\mathbf{N}^2(x=0,y) \right)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}x$$
$$= \begin{bmatrix} f_{b_1} \\ 0 \\ f_{b_4} \end{bmatrix}$$

$$q_{n} = 5 \text{ J/m}^{2} \text{s}$$

$$y \quad \text{Discretization}$$

$$k = 0.9 \text{ J/m}^{\circ} \text{C}$$

$$f = 2 \text{ J/m}^{2} \text{s}$$

$$h = 1 \text{ m}$$

$$q_{n} = 0$$

$$1$$

$$2$$

$$y \quad \text{Discretization}$$

$$k = 2 \text{ J/m}^{2} \text{s}$$

$$f = 2$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \end{split}$$



Assembly $\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix}$ -0.3380.000 0.000 0.338 $\mathbf{K} = \begin{bmatrix} -0.388 & 0.938\\ 0.000 & -0.600 \end{bmatrix}$ -0.6000.000 0.600 0.000 0.000 0.000 0.0000.000 $q_n = 5 \text{ J/m}^2 \text{s}$ Y Discretization 3 20 ° C $\begin{array}{l} k \,=\, 0.9 \,\, \mathrm{J/ms}^{\odot} \mathrm{C} \\ f \,=\, 2 \,\, \mathrm{J/m}^2 \mathrm{s} \\ h \,=\, 1 \,\, \mathrm{m} \end{array}$ 0 Ш dn 1 $q_n = 0$

$$\mathbf{K}^{1} = \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix}$$
$$\mathbf{K}^{2} = \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix}$$
$$\mathbf{f}^{1} = \mathbf{f}^{2} = \begin{bmatrix} 4\\ 4\\ 4\\ \end{bmatrix}$$
$$\mathbf{f}^{1}_{b} = \begin{bmatrix} 0\\ 0\\ 0\\ \end{bmatrix}, \ \mathbf{f}^{2}_{b} = \begin{bmatrix} f_{b_{1}}\\ 10\\ f_{b_{4}} + 10 \end{bmatrix}$$



Assembly $\mathbf{K}^2 = \begin{bmatrix} 0.600 & 0.000 \\ 0.000 & 0.338 \\ -0.600 & -0.338 \end{bmatrix}$ -0.600-0.3380.938 0.938 -0.3380.000 -0.600-0.388 0.000 0.938 -0.6000.000 $\mathbf{K} =$ -0.6000.938 -0.3380.6000.000-0.3380.938 $q_n = 5 \text{ J/m}^2 \text{s}$ Y Discretization 3 20 ° C $k\,=\,0.9\,\,\mathrm{J/ms}^{\rm O}\,\mathrm{C}$ 0 $f = 2 \text{ J/m}^2 \text{s}$ h = 1 mШ Ш 1n 1 $q_n = 0$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b_{1}} \\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \end{split}$$





$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b_{1}}\\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.338 & 0.338 \\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \end{split}$$





$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\kappa} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ 0.000 & -0.600 & 0.938 & -0.338 \\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \end{split}$$




$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b_{1}} \\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.630 & 0.038 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \end{split}$$



Assembly

$$\mathbf{f}_{b}^{1} = \begin{bmatrix} 0\\0\\0 \end{bmatrix} \qquad \mathbf{f}_{b}^{2} = \begin{bmatrix} f_{b1}\\10\\f_{b4}+10 \end{bmatrix}$$
$$\mathbf{f}_{b} = \begin{bmatrix} f_{b1}\\0\\f_{b4}+10 \end{bmatrix}$$
$$\mathbf{f}_{b} = \begin{bmatrix} f_{b1}\\0\\10\\f_{b4}+10 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b_{1}}\\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.938 & -0.338 & 0.000 & 0.000\\ 0.000 & -0.600 & 0.938 & -0.338\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \end{split}$$



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.938 & -0.338 & 0.000 & -0.600\\ 0.0388 & 0.338 & -0.338 & 0.000 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.938 & -0.338 & 0.038 & 0.000 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.938 & -0.600 & 0.938 & -0.338 \\ 0.600 & 0.000 & -0.338 & 0.338 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \end{split}$$

FEM equation set:
$$\mathbf{K}\mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$$

$$\begin{bmatrix} 0.938 - 0.338 & 0.000 - 0.600 \\ -0.388 & 0.938 - 0.600 & 0.000 \\ 0.000 - 0.600 & 0.938 - 0.338 \\ -0.600 & 0.000 - 0.338 & 0.938 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \Theta_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 8 \\ 4 \end{bmatrix} + \begin{bmatrix} f_{b_1} \\ 0 \\ 10 \\ f_{b_4} + 10 \end{bmatrix}$$



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.938 & -0.338 & 0.000 & -0.600\\ 0.038 & 0.938 & -0.338 & 0.038 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ 0.000 & -0.600 & 0.938 & -0.338\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \end{split}$$

FEM equation set:
$$\mathbf{K}\mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$$

$$\begin{bmatrix} 0.938 - 0.338 & 0.000 - 0.600 \\ -0.388 & 0.938 - 0.600 & 0.000 \\ 0.000 - 0.600 & 0.938 - 0.338 \\ -0.600 & 0.000 - 0.338 & 0.938 \end{bmatrix} \begin{bmatrix} 20 \\ \Theta_2 \\ \Theta_3 \\ 20 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 8 \\ 4 \end{bmatrix} + \begin{bmatrix} f_{b_1} \\ 0 \\ 10 \\ f_{b_4} + 10 \end{bmatrix}$$



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.338 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.600 & 0.938 & -0.338\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} \frac{8}{4}\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$

FEM equation set:
$$\mathbf{K}\mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$$

$$\begin{bmatrix} 0.938 - 0.338 & 0.000 - 0.600 \\ -0.388 & 0.938 - 0.600 & 0.000 \\ 0.000 - 0.600 & 0.938 - 0.338 \\ -0.600 & 0.000 - 0.338 & 0.938 \end{bmatrix} \begin{bmatrix} 20 \\ \Theta_2 \\ \Theta_3 \\ 20 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 8 \\ 4 \end{bmatrix} + \begin{bmatrix} f_{b_1} \\ 0 \\ 10 \\ f_{b_4} + 10 \end{bmatrix}$$
Solution: $\Theta_2 = 48.040, \Theta_3 = 57.145, f_{b_1} = -17.463, f_{b_4} = -26.537$

$$q_n = 5 \text{ J/m}^{2_5}$$

 $q_n = 0$



Heat flux vector - element 1

 $= 20^{\circ} C$

Ы

 $k = 0.9 \; \text{J}/$ $f = 2 \, \text{J/m}^2$ h = 1 m

 $q_n =$

$$\mathbf{\Theta}^1 = \begin{bmatrix} 20\\ 48.040\\ 57.145 \end{bmatrix}$$

$$q_n = 5 \text{ J/m}^2 \text{s}$$

$$y \text{ Discretization}$$

$$k = 0.9 \text{ J/ms}^\circ \text{C}$$

$$f = 2 \text{ J/m}^2 \text{s}$$

$$h = 1 \text{ m}$$

$$q_n = 0$$

$$1$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000 \\ -0.388 & 0.938 & -0.600 \\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600 \\ 0.000 & 0.338 & -0.338 \\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1} \\ 10 \\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\kappa} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600 \\ -0.388 & 0.938 & -0.600 & 0.000 \\ -0.388 & 0.938 & -0.630 & 0.000 \\ 0.000 & -0.600 & 0.938 & -0.338 \\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8 \\ 4 \\ 8 \\ 4 \\ \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 10 \\ f_{b4} + 10 \\ \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20 \\ 48.040 \\ 57.145 \\ 20 \end{bmatrix} \end{split}$$



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.038 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} \frac{8}{4}\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$

q



Heat flux vector – element 2

22222

 $k = 0.9 \, \downarrow$ f = 2 J/mh = 1 m

 $q_n =$

 $= 20^{\circ} C$

Ы

$$\Theta^2 = \begin{bmatrix} 20\\57.145\\20 \end{bmatrix}$$

$$q_n = 5 \text{ J/m}^2 \text{s}$$

$$y \qquad \text{Discretization}$$

$$k = 0.9 \text{ J/m}^2 \text{s}$$

$$k = 0.9 \text{ J/m}^2 \text{s}$$

$$h = 1 \text{ m}$$

$$q_n = 0$$

$$q_n = 0$$

$$q_n = 0$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\kappa} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ 0.000 & -0.600 & 0.938 & -0.338 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$



Heat flux vector – element 2

>>>>>>

 $k = 0.9 \; \text{J/ms}$ $f = 2 \text{ J/m}^2 \text{s}$ h = 1 m

 $q_n = 0$

 $= 20^{\circ} C$

Ы

$$\Theta^{2} = \begin{bmatrix} 20\\57.145\\20 \end{bmatrix}$$
$$\mathbf{q}^{2} = -k\mathbf{B}^{2}\Theta^{2}$$
$$= -0.9 \begin{bmatrix} 0.000 & 0.250 & -0.250\\-0.333 & 0.000 & 0.333 \end{bmatrix} \begin{bmatrix} 20\\57.145\\20 \end{bmatrix}$$
$$= \begin{bmatrix} -8.358\\0.000 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & 0.600 & -0.600\\ -0.000 & -0.600 & 0.938 & -0.338\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b1}\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{K} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.600 & 0.938 & -0.338\\ -0.600 & 0.000 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b1}\\ 0\\ 10\\ f_{b4} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$

$$\boldsymbol{\Theta}^{1} = \begin{bmatrix} 20\\ 48.040\\ 57.145 \end{bmatrix}$$
$$T^{e}(x^{e}, y^{e}) = \mathbf{N}^{e}(x^{e}, y^{e})\boldsymbol{\Theta}^{e}$$



$$\Theta^{1} = \begin{bmatrix} 20 \\ 48.040 \\ 57.145 \end{bmatrix}$$

$$F^{e}(x^{e}, y^{e}) = \mathbf{N}^{e}(x^{e}, y^{e})\Theta^{e}$$
e.g. at centre of mass $(\frac{8}{3}, 1)$

$$T^{1}\left(\frac{8}{3}, 1\right) = \begin{bmatrix} 1 - \frac{1}{4} \cdot \frac{8}{3} & \frac{1}{4} \cdot \frac{8}{3} - \frac{1}{3} \cdot 1 & \frac{1}{3} \cdot 1 \end{bmatrix} \begin{bmatrix} 20 \\ 48.040 \\ 57.145 \end{bmatrix}$$

$$F^{1}\left(\frac{8}{3}, 1\right) = \begin{bmatrix} 1 - \frac{1}{4} \cdot \frac{8}{3} & \frac{1}{4} \cdot \frac{8}{3} - \frac{1}{3} \cdot 1 & \frac{1}{3} \cdot 1 \end{bmatrix} \begin{bmatrix} 20 \\ 48.040 \\ 57.145 \end{bmatrix}$$

$$F^{1}_{b}$$

$$= 41.728$$
K



$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.338 & -0.338 & 0.000\\ -0.388 & 0.938 & -0.600\\ 0.000 & -0.600 & 0.600 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.600 & 0.000 & -0.600\\ 0.000 & 0.338 & -0.338\\ -0.600 & -0.338 & 0.938 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 4\\ 4\\ 4 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} 0\\ 0\\ 0\\ \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} f_{b_{1}} \\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \\ \mathbf{\kappa} &= \begin{bmatrix} 0.938 & -0.338 & 0.000 & -0.600\\ -0.388 & 0.938 & -0.600 & 0.000\\ -0.388 & 0.938 & -0.600 & 0.938 & -0.600\\ -0.600 & 0.000 & -0.338 & 0.338 \end{bmatrix} \\ \mathbf{f} &= \begin{bmatrix} 8\\ 4\\ 8\\ 4 \end{bmatrix}, \ \mathbf{f}_{b} &= \begin{bmatrix} f_{b_{1}} \\ 10\\ 10\\ f_{b_{4}} + 10 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20\\ 48.040\\ 57.145\\ 20 \end{bmatrix} \end{split}$$











$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma_{\mathrm{q}}} w h \widehat{q} \mathrm{d} \Gamma - \int_{\Gamma_{\mathrm{T}}} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

+ boundary condition

$$T=\widehat{T}$$
 on Γ_{T}



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T}$$
 on Γ_{T}



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\mathbf{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\mathbf{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\boldsymbol{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\boldsymbol{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \text{const}$$
$$\mathbf{w}^{\mathrm{T}}\int_{A}\mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \boldsymbol{\Theta} = -\mathbf{w}^{\mathrm{T}}\int_{\Gamma}\mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \mathbf{w}^{\mathrm{T}}\int_{A}\mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \forall \mathbf{w}$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\mathbf{\Theta}, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\mathbf{\Theta}$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \text{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}dA \ \mathbf{\Theta} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}d\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}fdA$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

$$T = \widehat{T} \quad \text{on } \Gamma_{\mathrm{T}}$$

$$T = \mathbf{N}\Theta, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\Theta$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \Theta = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A$$
$$\mathbf{K} = \int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A, \quad \mathbf{f} = \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \mathbf{f}_{b} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma$$



$$\int_{A} (\nabla w)^{\mathrm{T}} \mathbf{D} h \nabla T \mathrm{d} A = - \int_{\Gamma} w h q_{\mathrm{n}} \mathrm{d} \Gamma + \int_{A} w h f \mathrm{d} A$$

+ boundary condition

$$T = \widehat{T}$$
 on Γ_{T}

$$T = \mathbf{N}\Theta, \qquad w = \mathbf{N}\mathbf{w} = \mathbf{w}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}, \qquad \nabla T = \mathbf{B}\Theta$$
$$\nabla w = \mathbf{w}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}, \qquad \mathbf{D} = k\mathbf{I}, \qquad h = \mathrm{const}$$
$$\int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A \ \Theta = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma + \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A$$
$$\mathbf{K} = \int_{A} \mathbf{B}^{\mathrm{T}}k\mathbf{B}\mathrm{d}A, \quad \mathbf{f} = \int_{A} \mathbf{N}^{\mathrm{T}}f\mathrm{d}A, \quad \mathbf{f}_{b} = -\int_{\Gamma} \mathbf{N}^{\mathrm{T}}q_{\mathrm{n}}\mathrm{d}\Gamma$$
$$\mathbf{K}\Theta = \mathbf{f} + \mathbf{f}_{b}$$



 ${f K}$ matrix – element 1

$$\mathbf{N}^{1} = \left[\begin{array}{cc} \underline{(x^{(1)}-2)(y^{(1)}-3)} \\ 2\cdot3 \end{array} \begin{array}{c} \underline{x^{(1)}(y^{(1)}-3)} \\ -2\cdot3 \end{array} \begin{array}{c} \underline{x^{(1)}y^{(1)}} \\ 2\cdot3 \end{array} \begin{array}{c} \underline{(x^{(1)}-2)y^{(1)}} \\ -2\cdot3 \end{array} \right]$$



${\bf K}$ matrix – element 1

$$\mathbf{N}^{1} = \begin{bmatrix} \underline{(x^{(1)}-2)(y^{(1)}-3)} & \underline{x^{(1)}(y^{(1)}-3)} & \underline{x^{(1)}y^{(1)}} & \underline{(x^{(1)}-2)y^{(1)}} \\ 2\cdot3 & 2\cdot3 & 2\cdot3 & 2\cdot3 & -2\cdot3 \end{bmatrix}$$
$$\mathbf{B}^{1} = \nabla \mathbf{N}^{1} = \begin{bmatrix} \underline{y^{(1)}-3} & \underline{y^{(1)}-3} & \underline{y^{(1)}} & \underline{y^{(1)}} \\ 6 & -6 & \underline{x^{(1)}-2} \\ \underline{x^{(1)}-2} & \underline{x^{(1)}} & -\frac{x^{(1)}}{6} & \underline{x^{(1)}-2} \\ \end{bmatrix}$$





5

2

${\bf K}$ matrix – element 1

 $q_n = 0$

$$\mathbf{N}^{1} = \begin{bmatrix} \frac{(x^{(1)}-2)(y^{(1)}-3)}{2\cdot3} & \frac{x^{(1)}(y^{(1)}-3)}{2\cdot3} & \frac{x^{(1)}y^{(1)}}{2\cdot3} & \frac{(x^{(1)}-2)y^{(1)}}{-2\cdot3} \end{bmatrix}$$
$$\mathbf{B}^{1} = \nabla \mathbf{N}^{1} = \begin{bmatrix} \frac{y^{(1)}-3}{6} & \frac{y^{(1)}-3}{-6} & \frac{y^{(1)}}{6} & \frac{y^{(1)}}{-6} \\ \frac{x^{(1)}-2}{-6} & \frac{x^{(1)}}{-6} & \frac{x^{(1)}}{6} & \frac{x^{(1)}-2}{-6} \end{bmatrix}$$
$$\mathbf{K}^{1} = \int_{A^{1}} \mathbf{B}^{T} k \mathbf{B} dA = \int_{0}^{3} \int_{0}^{2} \mathbf{B}^{T} k \mathbf{B} dx^{(1)} dy^{(1)} =$$
$$= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ 0.025 & -0.325 & 0.025 & 0.650 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix}$$
$$\overset{\mathbf{y}_{11}}{\overset{\mathbf{y}_{12}}{\overset{\mathbf{y}_{12}}{\overset{\mathbf{y}_{11}}{\overset{\mathbf{y}_{12}$$

 $q_{n} = 0$

K matrix – element 2 0.650 - 0.350 - 0.3250.025 $\mathbf{K}^{1} = \begin{bmatrix} -0.350 & 0.650 & 0.025 & -0.325 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix}$ $\mathbf{N}^{2} = \left| \begin{array}{c} \frac{(x^{2}-2)(y^{2}-3)}{2\cdot 3} & \frac{x^{2}(y^{2}-3)}{-2\cdot 3} & \frac{x^{2}(y^{2})}{2\cdot 3} & \frac{(x^{2}-2)y^{2}}{-2\cdot 3} \end{array} \right|$ $q_n = \sigma_{J/m}$ s $y^{(2)}$ >>>>>>>>>>> 3 = 20 ° C $k = 0.9 \text{ J/ms}^{\circ}\text{C}$ $f = 2 \text{ J/m}^{2}\text{s}$ h = 1 m



${f K}$ matrix – element 2

$$\mathbf{N}^{2} = \begin{bmatrix} \underline{(x^{(2)}-2)(y^{(2)}-3)} & \underline{x^{(2)}(y^{(2)}-3)} & \underline{x^{(2)}y^{(2)}} & \underline{(x^{(2)}-2)y^{(2)}} \\ \hline \mathbf{B}^{2} = \nabla \mathbf{N}^{2} = \begin{bmatrix} \underline{y^{(2)}-3} & \underline{y^{(2)}-3} & \underline{y^{(2)}} & \underline{y^{(2)}} \\ \underline{6} & \underline{-6} & \underline{6} & \underline{-6} \\ \underline{x^{(2)}-2} & \underline{-6} & \underline{-6} & \underline{-6} \end{bmatrix}$$

$$\mathbf{K}^1 = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 - 0.350 & 0.650 \end{bmatrix}$$





${f K}$ matrix – element 2

 $= 20^{\circ} C$

F

 $k = 0.9 \text{ J/ms}^{\circ}\text{C}$ $f = 2 \text{ J/m}^{2}\text{s}$

h = 1 m $q_n = 0$ $q_n = 0$

$$\mathbf{N}^{2} = \begin{bmatrix} \frac{(x^{2}-2)(y^{2}-3)}{2\cdot3} & \frac{x^{2}(y^{2}-3)}{2\cdot3} & \frac{x^{2}(y^{2})}{2\cdot3} & \frac{(x^{2}-2)y^{2}}{-2\cdot3} \end{bmatrix}$$
$$\mathbf{B}^{2} = \nabla \mathbf{N}^{2} = \begin{bmatrix} \frac{y^{2}-3}{6} & \frac{y^{2}-3}{-6} & \frac{y^{2}}{6} & \frac{y^{2}}{-6} \\ \frac{x^{2}-2}{6} & \frac{x^{2}}{-6} & \frac{x^{2}}{6} & \frac{x^{2}-2}{-6} \end{bmatrix}$$
$$\mathbf{K}^{2} = \int_{A^{2}} \mathbf{B}^{\mathrm{T}} k \mathbf{B} \mathrm{d} A = \int_{0}^{3} \int_{0}^{2} \mathbf{B}^{\mathrm{T}} k \mathbf{B} \mathrm{d} x^{2} \mathrm{d} y^{2} = \\ = \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix}$$

$$\mathbf{K}^1 = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix}$$





f vector – element 1 and 2 -
$$A^1 = A^2$$

$$\mathbf{f}^e = \int_{A^e} \mathbf{N}^{\mathrm{T}} f \mathrm{d}A = \frac{f}{4} A^e \begin{bmatrix} 1\\1\\1\\1 \end{bmatrix} = \begin{bmatrix} 3\\3\\3\\3 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.356 \\ 0.025 - 0.325 & 0.350 & 0.650 \\ \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 & 0.055 \\ 0.025 - 0.325 & 0.0350 & 0.650 \\ \end{bmatrix}$$





\mathbf{f}_b vector – element 1

= 20 ° C

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$$\begin{split} \mathbf{f}_{b}^{1} &= -\int_{\Gamma_{ij}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \\ &- \int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{li}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \end{split}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ 0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 & -0.350 & 0.650 & -0.350 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \end{split}$$





\mathbf{f}_b vector – element 1

$$\mathbf{f}_{b}^{1} = -\int_{\Gamma_{ij}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} \mathbf{q}_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

flow continuity along edge 5-6 $q_{njk}^{1} = -q_{nli}^{2}$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & -0.325 & 0.025 \\ -0.325 & 0.025 & -0.650 & -0.326 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \end{split}$$







\mathbf{f}_b vector – element 1

$$\mathbf{f}_{b}^{1} = -\int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{li}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.350 & 0.6650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & 0.0350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \end{split}$$



$$\begin{array}{c} q_n = 5 \; \mathsf{J/m^2s} \\ \begin{array}{c} \searrow \searrow \bigcirc \searrow \bigcirc \searrow \bigcirc \searrow \bigcirc \searrow \bigcirc \searrow \bigcirc \swarrow \\ k = 0.9 \; \mathsf{J/ms^oC} \\ f = 2 \; \mathsf{J/m^2s} \\ h = 1 \; \mathsf{m} \\ \end{array} \\ \begin{array}{c} q_n = 0 \end{array} \end{array}$$



$$\mathbf{f}_{b}^{1} = -\int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{li}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$
$$-\int_{\Gamma_{kl}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma = -\int_{0}^{2} (\mathbf{N}^{1} (x^{(1)}, y^{(1)} = 3))^{\mathrm{T}} (-5) \mathrm{d}x^{(1)}$$
$$= \begin{bmatrix} 0\\0\\5\\5\\5 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 - 0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & 0.025 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \end{split}$$



$$\mathbf{f}_{b}^{1} = \begin{bmatrix} 0\\0\\5\\5 \end{bmatrix} - \int_{\Gamma_{li}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$
$$- \int_{\Gamma_{li}^{1}} (\mathbf{N}^{1})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma = -\int_{0}^{3} (\mathbf{N}^{1} (x^{(1)} = 0, y^{(1)}))^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}y^{(1)}$$
$$= \begin{bmatrix} f_{b1}\\0\\0\\f_{b4}\end{bmatrix}$$
$$\overset{(n \to 0)}{\underset{k \to 0}{\text{ for } 3}} \overset{(n \to 0)}{\underset{k \to 0}{\overset{(n \to 0)}{\underset{k \to 0}{\text{ for } 3}}} \overset{(n \to 0)}{\underset{$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \\ \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & 0.650 \\ -0.325 & 0.025 & 0.650 - 0.325 \\ 0.025 - 0.325 & 0.0350 & 0.650 \\ \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \end{split}$$



$$\mathbf{f}_b^1 = \begin{bmatrix} 0\\0\\5\\5 \end{bmatrix} + \begin{bmatrix} f_{b1}\\0\\0\\f_{b4} \end{bmatrix}$$
$$\mathbf{f}_b^1 = \begin{bmatrix} f_{b1}\\0\\5\\5+f_{b4} \end{bmatrix}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 & 0.350 \\ 0.025 - 0.325 & 0.050 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \end{split}$$





\mathbf{f}_b vector – element 2

$$\begin{split} \mathbf{f}_{b}^{2} &= -\int_{\Gamma_{ij}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \\ &- \int_{\Gamma_{kl}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{li}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \end{split}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.025 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & 0.0350 & 0.025 & 0.025 \\ -0.350 & 0.650 & 0.025 & 0.025 & 0.025 \\ 0.0325 & 0.025 & 0.650 & -0.335 \\ 0.025 - 0.325 & 0.0350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} fb_1 \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \end{split}$$





$$\begin{aligned} \mathbf{f}_{b}^{2} &= -\int_{\Gamma_{ij}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} \frac{q_{\mathrm{n}}}{q_{\mathrm{n}}} \mathrm{d}\Gamma - \int_{\Gamma_{jk}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} \frac{q_{\mathrm{n}}}{q_{\mathrm{n}}} \mathrm{d}\Gamma \\ &- \int_{\Gamma_{kl}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma - \int_{\Gamma_{li}^{2}} (\mathbf{N}^{2})^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma \\ & \text{flow continuity} \\ & \text{along edge 5-6} \\ & q_{njk}^{1} = -q_{nli}^{2} \end{aligned}$$



$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & 0.650 & -0.350 \\ 0.025 - 0.325 & 0.650 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \end{split}$$



$$\mathbf{f}_b^2 = -\int_{\Gamma_{kl}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.025 & 0.025 & 0.035 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \end{split}$$




\mathbf{f}_b vector – element 2

 $q_n = 5 \text{ J/m}^2 \text{s}$

 $k = 0.9 \text{ J/ms}^{\circ}\text{C}$ $f = 2 \text{ J/m}^{2}\text{s}$

 $q_n = 0$

h = 1 m

 $= 20^{\circ} C$

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$$\mathbf{f}_b^2 = -\int_{\Gamma_{kl}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma$$

$$-\int_{\Gamma_{kl}^2} (\mathbf{N}^2)^{\mathrm{T}} q_{\mathrm{n}} \mathrm{d}\Gamma = -\int_0^2 (\mathbf{N}^1(x^{(2)}, y^{(2)} = 3))^{\mathrm{T}} (-5) \mathrm{d}x^{(2)}$$
$$\begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix}$$

 $= \begin{bmatrix} 5\\5\\5 \end{bmatrix}$

 $q_{n} = 0$

$$\begin{array}{c} 4 \\ y^{(2)} \\ 1 \\ y^{(2)} \\ y$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.650 - 0.350 & -0.350 & 0.650 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \end{split}$$



\mathbf{f}_b vector – element 2

$$\mathbf{f}_b^2 = \begin{bmatrix} 0\\0\\5\\5 \end{bmatrix}$$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.025 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & 0.035 \\ -0.350 & 0.650 & 0.025 & 0.025 \\ -0.325 & 0.025 & 0.650 - 0.330 \\ 0.025 - 0.325 & 0.050 & -0.350 \\ 0.025 - 0.325 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \end{split}$$



Assembly				
$\mathbf{K}^1 = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{cccc} 5 & 0.025 & $	
		5 6	4	
	350 0.000	0.000 0.	025 - 0.350	-0.325
		0.000 0.		0.000
$\mathbf{K} = \begin{bmatrix} 0.0\\ 0.0 \end{bmatrix}$	0.000	0.000 0.	650 - 0.325	-0.350
-0.3	0.000	0.000 -0.	325 0.650	0.025
	325 0.000	0.000 - 0.	350 0.025	0.650

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.650 - 0.350 & 0.025 & -0.325 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & -0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^{2}_{b} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \end{split}$$







Assen	nbly						
$\mathbf{K}^2 =$	$\left[\begin{array}{c} 0.65\\ -0.35\\ -0.32\\ 0.02\end{array}\right]$	$ \begin{array}{r} 50 & -0. \\ 50 & 0. \\ 25 & 0. \\ 25 & -0. \\ \end{array} $	$\begin{array}{cccc} 350 & -0 \\ 650 & 0 \\ 025 & 0 \\ 325 & -0 \end{array}$).325).025 -).650 -).350	$0.025 \\ -0.325 \\ -0.350 \\ 0.650 $	5 2 3 6	
(5) (2) (3) (6)							
	0.650	0.000	0.000	0.025	-0.350	-0.325	
	0.000	0.650	0.025	0.000	-0.350	-0.325	
K =	0.000	0.025	0.650	0.000	-0.325	-0.350	
	0.025	0.000	0.000	0.650	-0.325	-0.350	
	-0.350	-0.350	-0.325	-0.325	1.300	0.050	
	-0.325	-0.325	-0.350	-0.350	0.050	1.300	

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.0350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.350 & 0.650 \\ -0.350 & 0.650 & 0.025 & 0.025 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ 0.025 - 0.325 & 0.0350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^2_b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \end{split}$$









$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \\ \end{bmatrix} \\ \mathbf{K}^{2} &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & -0.325 & 0.025 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ 0.025 - 0.325 & -0.350 & 0.650 \\ \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \quad \mathbf{f}^{2}_{b} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ 5 \end{bmatrix} \end{split}$$







Assembly



$$\begin{split} \mathbf{K}^1 &= \left[\begin{array}{c} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \\ \end{array} \right] \\ \mathbf{K}^2 &= \left[\begin{array}{c} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & -0.325 \\ -0.325 & 0.025 & -0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \\ \end{array} \right] \\ \mathbf{f}^1 &= \mathbf{f}^2 = \left[\begin{array}{c} 3 \\ 3 \\ 3 \\ 3 \\ \end{array} \right] \\ \mathbf{f}^1_b &= \left[\begin{array}{c} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{array} \right], \ \mathbf{f}^2_b = \left[\begin{array}{c} 0 \\ 0 \\ 5 \\ 5 \\ \end{array} \right] \end{split}$$

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h = 1 mы

 $q_n = 0$

$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & 0.650 & 0.055 \\ -0.350 & 0.650 & 0.025 & 0.025 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & 0.650 & -0.350 \\ 0.025 - 0.325 & 0.650 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^2_b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \end{split}$$



$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^2_b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ \end{bmatrix} \end{split}$$

FEM equation set: $\mathbf{K} \mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$

$$\begin{bmatrix} 0.650 & 0.000 & 0.000 & 0.025 & -0.350 & -0.325 \\ 0.000 & 0.650 & 0.025 & 0.000 & -0.350 & -0.325 \\ 0.025 & 0.000 & 0.050 & 0.650 & -0.325 & -0.350 \\ -0.325 & -0.325 & -0.325 & -0.325 & 1.300 & 0.550 \\ -0.325 & -0.325 & -0.350 & -0.350 & 0.050 & 1.300 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_5 \\ \theta_6 \\ \theta_6 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} f_{b1} \\ 5 \\ 5 + f_{b4} \\ 0 \\ 10 \end{bmatrix}$$





$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ -0.325 & 0.025 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^2_b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ \end{bmatrix} \end{split}$$

FEM equation set: $\mathbf{K} \mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$

$$\begin{bmatrix} 0.650 & 0.000 & 0.000 & 0.025 & -0.350 & -0.325 \\ 0.000 & 0.625 & 0.000 & -0.350 & -0.325 \\ 0.025 & 0.000 & 0.000 & 0.650 & -0.325 & -0.350 \\ -0.350 & -0.350 & -0.325 & -0.350 & 0.050 & 1.300 \end{bmatrix} \begin{bmatrix} 20 \\ \Theta_3 \\ \Theta_3 \\ \Theta_6 \\ \Theta_6 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \\ 0 \\ 10 \end{bmatrix}$$





FEM equation set: $\mathbf{K} \mathbf{\Theta} = \mathbf{f} + \mathbf{f}_b$

$$\begin{bmatrix} 0.650 & 0.000 & 0.000 & 0.025 & -0.350 & -0.325 \\ 0.000 & 0.650 & 0.025 & 0.000 & -0.350 & -0.325 \\ 0.025 & 0.000 & 0.000 & 0.650 & -0.325 & -0.350 \\ -0.325 & -0.325 & -0.325 & -0.325 & 1.300 & 0.050 \\ -0.325 & -0.325 & -0.350 & -0.350 & 0.050 & 1.300 \end{bmatrix} \begin{bmatrix} 20 \\ \Theta_2 \\ \Theta_3 \\ \Theta_6 \\ \Theta_6 \\ \Theta_6 \\ \Theta_6 \\ \Theta_6 \\ \Theta_6 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 \\ + f_{b4} \\ 10 \end{bmatrix}$$

elution: $\Theta_2 = 48.429, \ \Theta_3 = 56.756, \ \Theta_5 = 40.361, \ \Theta_6 = 48.528, \\ b_1 = -19.398, \ f_{b4} = -24.602$



$$\begin{split} \mathbf{K}^1 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.325 \\ -0.325 & 0.025 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{K}^2 &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^1 &= \mathbf{f}^2 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^1_b &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^2_b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ 5 \end{bmatrix} \end{split}$$



$\mathbf{K}^1 = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix}$ $\mathbf{K}^2 = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & 0.350 & 0.650 \end{bmatrix}$ $\mathbf{f}^1 = \mathbf{f}^2 = \begin{bmatrix} 3\\ 3\\ 3 \end{bmatrix}$ $\mathbf{f}_{b}^{1} = \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b} \end{bmatrix}, \ \mathbf{f}_{b}^{2} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix}$ $\Theta = \begin{bmatrix} 20\\ 48.429\\ 56.756\\ 20\\ 40.361 \end{bmatrix}$

 $= 20^{\circ} C$

 $q_n = 0$

$$\mathbf{\Theta}^{1} = \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$





Heat flux vector – element 1

$$\Theta^{1} = \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$
$$\mathbf{q}^{1} = -k\mathbf{B}^{1}\Theta^{1}$$
$$= -0.9 \begin{bmatrix} \frac{y^{(1)}-3}{6} & \frac{y^{(1)}-3}{-6} & \frac{y^{(1)}}{6} & \frac{y^{(1)}}{-6}\\ \frac{x^{(1)}-2}{6} & \frac{x^{(1)}}{-6} & \frac{x^{(1)}-2}{-6} \end{bmatrix} \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} -1.225y - 9.192\\ -1.225x \end{bmatrix}$$
e.g. at centre of mass $\mathbf{q}^{1}(1, 1.5) = \begin{bmatrix} -11.000\\ -1.225 \end{bmatrix}$

$$q_n = 5 J/m^{2s}$$

$$y^{(1)}_{4} = 6$$

$$k = 0.9 J/m^{s} C$$

$$k = 0.9 J/m^{s} C$$

$$f = 2 J/m^{2s}$$

$$h = 1 m$$

$$q_n = 0$$

$$q_n = 0$$

$$q_n = 0$$

$$\begin{split} \mathbf{\kappa}^{1} &= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & -0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{\kappa}^{2} &= \begin{bmatrix} 0.650 & -0.350 & 0.325 & -0.225 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.336 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20 \\ 48.429 \\ 248.429 \\ 48.528 \end{bmatrix} \end{split}$$



$\mathbf{K^1} = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix}$
$\kappa^2 = \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix}$
$\mathbf{f}^1 = \mathbf{f}^2 = \begin{bmatrix} 3\\ 3\\ 3\\ 3 \end{bmatrix}$
$\mathbf{f}_b^1 = \left[\begin{array}{c} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{array} \right], \ \ \mathbf{f}_b^2 = \left[\begin{array}{c} 0 \\ 0 \\ 5 \\ 5 \end{array} \right]$
$\boldsymbol{\Theta} = \begin{bmatrix} 20\\ 48.429\\ 56.756\\ 20\\ 40.361\\ 48.528 \end{bmatrix}$

Heat flux vector – element 2
$$\Theta^2 = \begin{bmatrix} 40.361\\ 48.429\\ 56.756\\ 48.528 \end{bmatrix}$$





Heat flux vector – element 2

$$\Theta^{2} = \begin{bmatrix} 40.361\\48.429\\56.756\\48.528 \end{bmatrix}$$
$$\mathbf{q}^{2} = -k\mathbf{B}^{2}\Theta^{2}$$
$$= -0.9 \begin{bmatrix} \frac{y^{(2)}-3}{6} & \frac{y^{(2)}}{-6} & \frac{y^{(2)}}{6} & \frac{y^{(2)}}{-6} \\ \frac{x^{(2)}-2}{6} & \frac{x^{(2)}}{-6} & \frac{x^{(2)}}{-6} \end{bmatrix} \begin{bmatrix} 40.361\\48.429\\56.756\\48.528 \end{bmatrix}$$
$$= \begin{bmatrix} -0.024y - 3.631\\-0.024x - 2.450 \end{bmatrix}$$
e.g. at centre of mass $\mathbf{q}^{2}(1, 1.5) = \begin{bmatrix} -3.667\\-2.474 \end{bmatrix}$



$$\begin{split} \mathbf{\kappa}^{1} &= \begin{bmatrix} 0.650 - 0.350 - 0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 - 0.325 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{\kappa}^{2} &= \begin{bmatrix} 0.650 - 0.350 & 0.035 & 0.025 \\ -0.350 & 0.650 & 0.025 & 0.025 \\ -0.325 & 0.025 & 0.650 - 0.350 \\ 0.025 - 0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \quad \mathbf{f}^{2}_{b} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20 \\ 48.429 \\ -48.528 \\ -40.361 \\ 40.361 \\ 48.528 \end{bmatrix} \end{split}$$



Computation of temperature at a point in el. 1

$$\boldsymbol{\Theta}^{1} = \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$
$$\boldsymbol{\nabla}^{e}(x^{e} \ y^{e}) = \boldsymbol{N}^{e}(x^{e} \ y^{e})\boldsymbol{\Theta}^{e}(x^{e} \ y^{e})\boldsymbol$$

$$\begin{split} \mathbf{K}^{1} &= \begin{bmatrix} 0.650 - 0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 - 0.325 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \\ \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \\ \boldsymbol{\Theta} &= \begin{bmatrix} 20 \\ 48.429 \\ 56.756 \\ 20 \\ 40.361 \\ 48.528 \end{bmatrix} \end{split}$$

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Computation of temperature at a point in el. 1

$$\Theta^{1} = \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$

$$T^{e}(x^{e}, y^{e}) = \mathbf{N}^{e}(x^{e}, y^{e})\Theta^{e}$$
e.g. at centre of mass (1, 1.5)

$$T^{1}(1, 1.5) = \begin{bmatrix} \frac{(1-2)(1.5-3)}{6} & \frac{1(1.5-3)}{-6} & \frac{-11.5}{6} & \frac{(1-2)1.5}{-6} \end{bmatrix} \begin{bmatrix} 20\\ 40.361\\ 48.528\\ 20 \end{bmatrix}$$

$$= 32.222$$





$$\begin{split} \mathbf{\kappa}^{1} &= \begin{bmatrix} 0.650 & -0.350 & -0.325 & 0.025 \\ -0.350 & 0.650 & 0.025 & -0.325 \\ -0.325 & 0.025 & -0.350 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{\kappa}^{2} &= \begin{bmatrix} 0.650 & -0.350 & -0.350 \\ -0.350 & 0.650 & -0.025 & -0.325 \\ -0.325 & 0.025 & 0.650 & -0.350 \\ 0.025 & -0.325 & -0.350 & 0.650 \end{bmatrix} \\ \mathbf{f}^{1} &= \mathbf{f}^{2} &= \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \\ \mathbf{f}^{1}_{b} &= \begin{bmatrix} f_{b1} \\ 0 \\ 5 \\ 5 + f_{b4} \end{bmatrix}, \ \mathbf{f}^{2}_{b} &= \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \end{bmatrix} \\ \mathbf{\Theta} &= \begin{bmatrix} 20 \\ 48.429 \\ 56.756 \\ 20 \\ 40.361 \\ 48.528 \end{bmatrix} \end{split}$$

