Frame statics solved using CALFEM toolbox for MATLAB

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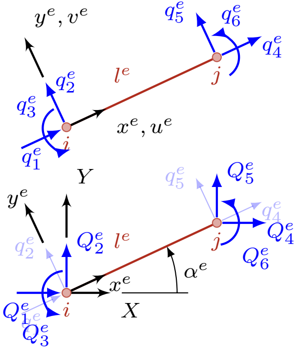
Frame element description

Approximation

$$\mathbf{u}^e(x) = \mathbf{N}^e(x)\mathbf{q}^e$$

$$\mathbf{N}^e = \begin{bmatrix} L_i^e & 0 & 0 & L_j^e & 0 & 0 \\ 0 & H_i^e & \widehat{H}_i^e & 0 & H_j^e & \widehat{H}_j^e \end{bmatrix}, \mathbf{q}^e = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$



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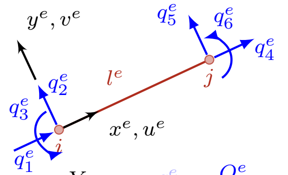

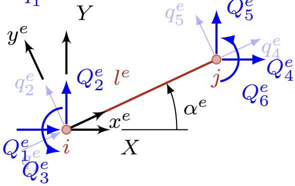

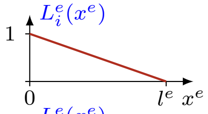

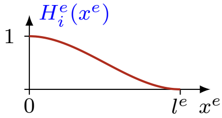

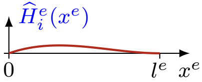

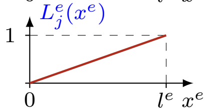

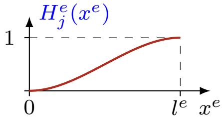

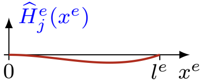

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Frame element description

Displacements vector

$$\mathbf{u} = \{u(x), v(x)\}$$

Strain vector

$$\mathbf{e} = \{\varepsilon_x, \kappa\}$$

Stress vector

$$\mathbf{s} = \{N(x), M(x)\}$$

Distributed loading

$$\mathbf{p} = \{p_x, p_y\}$$

Matrix of constitutive relationships

$$\mathbf{D} = \begin{bmatrix} EA & 0 \\ 0 & EI \end{bmatrix}$$

Differential operator matrix

$$\mathbf{L} = \begin{bmatrix} \dfrac{\mathrm{d}}{\mathrm{d}x} & 0 \\ 0 & -\dfrac{\mathrm{d}^2}{\mathrm{d}x^2} \end{bmatrix}$$

Kinematic and constitutive relations

$$\mathbf{e} = \mathbf{Lu} = \mathbf{LNq} = \mathbf{Bq}, \qquad \mathbf{s} = \mathbf{De} = \mathbf{DBq}$$

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Frame element description

Element stiffness matrix

$$\mathbf{k}^e = \int_0^{l^e} \mathbf{B}^{e\mathrm{T}} \mathbf{D}^e \mathbf{B}^e \mathrm{d}x^e$$

$$\mathbf{k}^e = \frac{EI}{l^3} \begin{bmatrix} \frac{Al^2}{I} & 0 & 0 & -\frac{Al^2}{I} & 0 & 0 \\ 0 & 12 & 6l & 0 & -12 & 6l \\ 0 & 6l & 4l^2 & 0 & -6l & 2l^2 \\ -\frac{Al^2}{I} & 0 & 0 & \frac{Al^2}{I} & 0 & 0 \\ 0 & -12 & -6l & 0 & 12 & -6l \\ 0 & 6l & 2l^2 & 0 & -6l & 4l^2 \end{bmatrix}^e$$

Transformation matrix

$c = \cos(\alpha^e)$ i $s = \sin(\alpha^e)$

$$\mathbf{T}^e = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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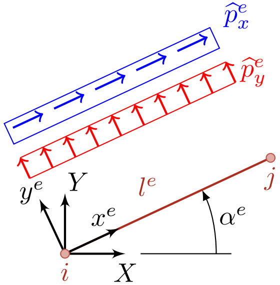
Frame element description

Substitute nodal forces

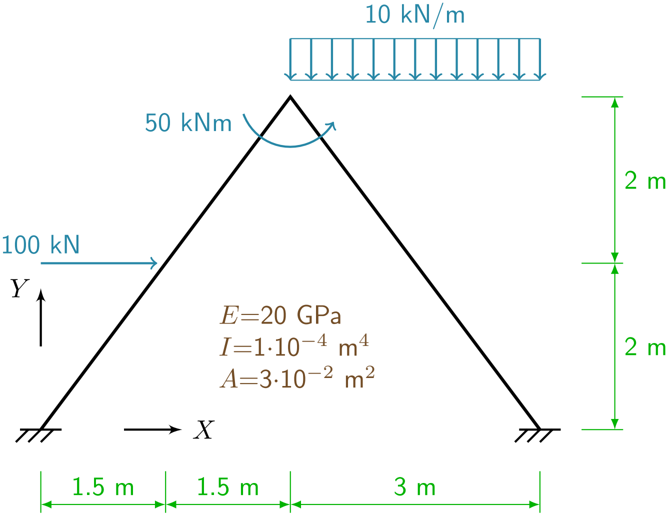
$$\mathbf{z}^e = \int_0^{l^e} \mathbf{N}^{e\mathrm{T}} \mathbf{p}^e \mathrm{d}x^e$$

for $p_x = \text{const} = \widehat{p}_x$ i $p_y = \text{const} = \widehat{p}_y$

$$\mathbf{z}^e = \left\{ \frac{\widehat{p}_x l}{2}, \ \frac{\widehat{p}_y l}{2}, \ \frac{\widehat{p}_y l^2}{12}, \ \frac{\widehat{p}_x l}{2}, \ \frac{\widehat{p}_y l}{2}, \ -\frac{\widehat{p}_y l^2}{12} \right\}^e$$



Assignment



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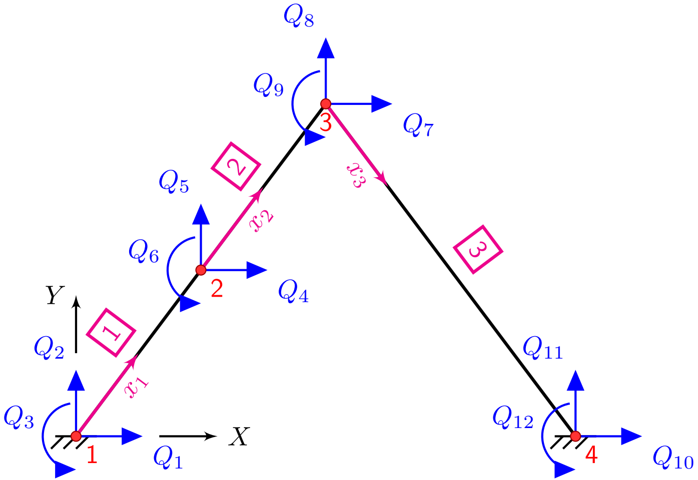
Discretization



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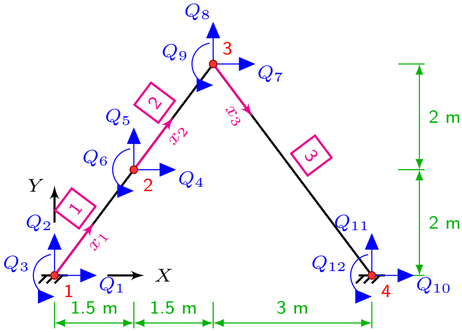
Script - *frame*.m

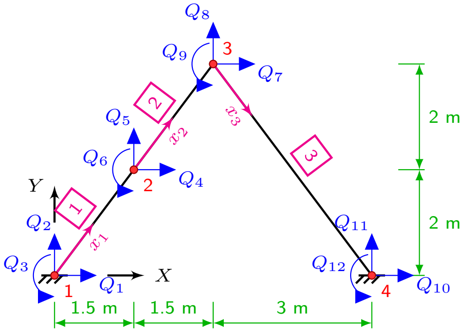
function frame()



Script - *frame*.m

```
function frame()

% definition of dof matrix
% for elements
Edof=[1 1 2 3 4 5 6 ;
      2 4 5 6 7 8 9;
      3 7 8 9 10 11 12];
```



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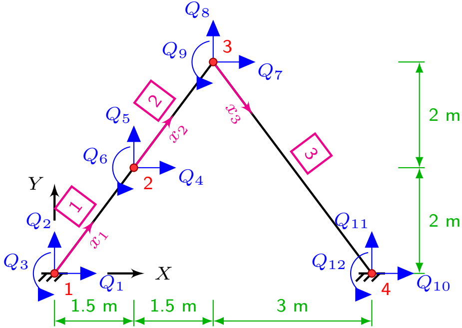
% definition of dof matrix
% for elements
Edof=[1 1 2 3 4 5 6 ;
      2 4 5 6 7 8 9;
      3 7 8 9 10 11 12];

% matrix of node coordinates
Coord=[0 0; 1.5 2; 3 4; 6 0];
```



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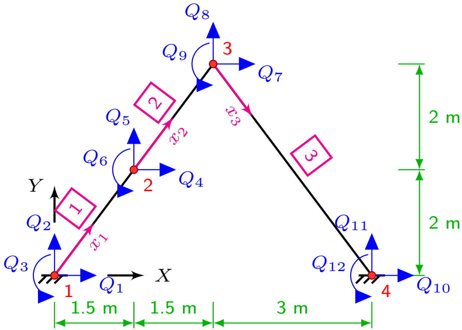

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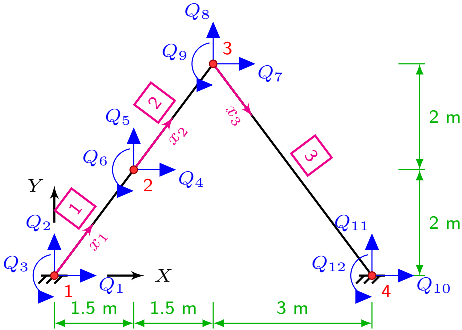
Edof=[1 1 2 3 4 5 6 ;
      2 4 5 6 7 8 9;
      3 7 8 9 10 11 12];

% matrix of node coordinates
Coord=[0 0; 1.5 2; 3 4; 6 0];

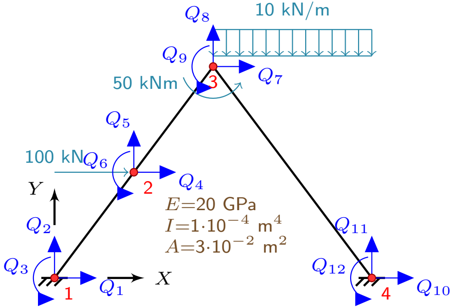
% matrix of dofs
Dof=[1 2 3; 4 5 6;
     7 8 9; 10 11 12];

% compute coordinate vectors
% for elements
[Ex,Ey]=coordxtr(Edof,Coord,Dof,2);
```



Script - *frame*.m

```
% material/section properties
E=2e7;
I=1e-4;
A=0.03;
ep=[E,A,I];
```



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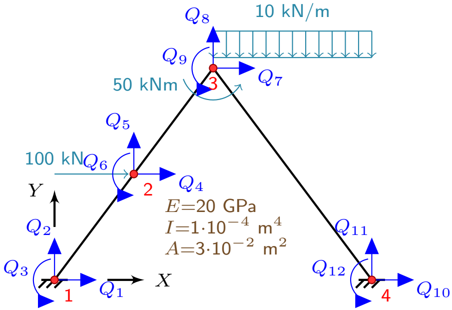
Script - *frame*.m

```
% material/section properties
E=2e7;
I=1e-4;
A=0.03;
ep=[E,A,I];

% zero global matrices
K=zeros(12);
F=zeros(12,1);
```



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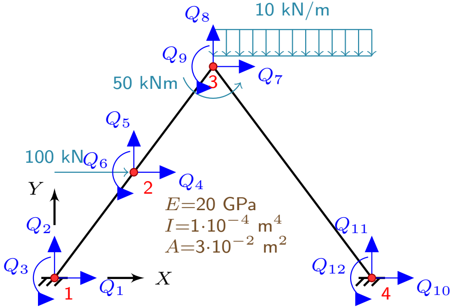
Script - *frame*.m

```
% material/section properties
E=2e7;
I=1e-4;
A=0.03;
ep=[E,A,I];

% zero global matrices
K=zeros(12);
F=zeros(12,1);

% account for
% concentrated loads
F(4)=100;
F(9)=50;
```



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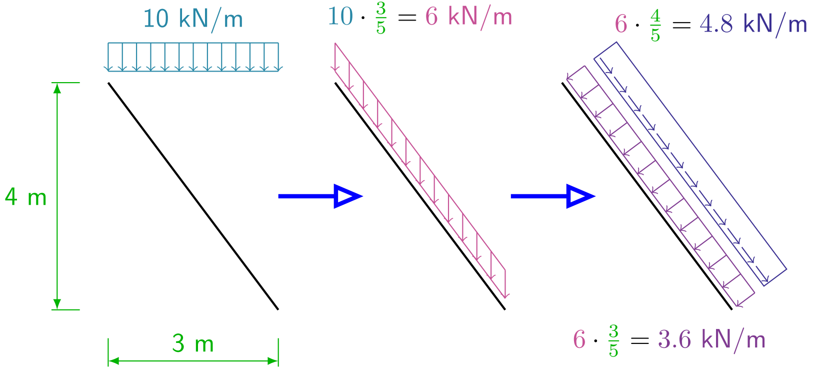
Equivalent nodal forces



eq=[4.8,-3.6]

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Script - *frame*.m

```
% plot the frame
eldraw2(Ex,Ey, [1,2,2]);
```

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Script - *frame*.m

```
% plot the frame
eldraw2(Ex,Ey, [1,2,2]);

% compute stiffness matrices for elements
Ke1=beam2e(Ex(1,:),Ey(1,:),ep);
Ke2=beam2e(Ex(2,:),Ey(2,:),ep);
[Ke3,Ze3]=beam2e(Ex(3,:),Ey(3,:),ep,eq);
```

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Script - *frame*.m

```
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eldraw2(Ex,Ey, [1,2,2]);

% compute stiffness matrices for elements
Ke1=beam2e(Ex(1,:),Ey(1,:),ep);
Ke2=beam2e(Ex(2,:),Ey(2,:),ep);
[Ke3,Ze3]=beam2e(Ex(3,:),Ey(3,:),ep,eq);

% assemble global stiffness matrix and load vector
K=assem(Edof(1,:),K,Ke1);
K=assem(Edof(2,:),K,Ke2);
[K,F]=assem(Edof(3,:),K,Ke3,F,Ze3);
```

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Script - *frame*.m

```
% plot the frame
eldraw2(Ex,Ey, [1,2,2]);

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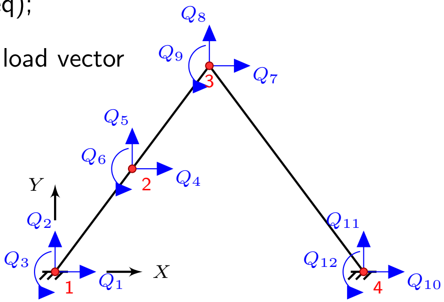
Ke1=beam2e(Ex(1,:),Ey(1,:),ep);
Ke2=beam2e(Ex(2,:),Ey(2,:),ep);
[Ke3,Ze3]=beam2e(Ex(3,:),Ey(3,:),ep,eq);

% assemble global stiffness matrix and load vector
K=assem(Edof(1,:),K,Ke1);
K=assem(Edof(2,:),K,Ke2);
[K,F]=assem(Edof(3,:),K,Ke3,F,Ze3);

% account for boundary conditions
bc=[ 1 0; 2 0; 3 0; 10 0; 11 0; 12 0];
```



Script - *frame*.m

```
% compute displacement and reaction force vector
[Q,R ]=solveq(K,F,bc)
```

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Script - *frame*.m

```
% compute displacement and reaction force vector
[Q,R ]=solveq(K,F,bc)

% extract nodal displacements for all elements
Qe=extract(Edof,Q);
```

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Script - *frame*.m

```
% compute displacement and reaction force vector
[Q,R ]=solveq(K,F,bc)

% extract nodal displacements for all elements
Qe=extract(Edof,Q);

% return to elements to compute nodal forces
f1=beam2s(Ex(1,:),Ey(1,:),ep,Qe(1,:))
f2=beam2s(Ex(2,:),Ey(2,:),ep,Qe(2,:))
f3=beam2s(Ex(3,:),Ey(3,:),ep,Qe(3,:),eq)
```

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Script - *frame*.m

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[Q,R ]=solveq(K,F,bc)

% extract nodal displacements for all elements
Qe=extract(Edof,Q);

% return to elements to compute nodal forces
f1=beam2s(Ex(1,:),Ey(1,:),ep,Qe(1,:))
f2=beam2s(Ex(2,:),Ey(2,:),ep,Qe(2,:))
f3=beam2s(Ex(3,:),Ey(3,:),ep,Qe(3,:),eq)

% draw deformed frame
eldisp2(Ex,Ey,Qe,[1,4,1]);
```

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Results

Q=

0
0
0
0.0355
-0.0264
-0.0073
0.0003
-0.0001
0.0288
0
0
0

R=

-69.3890
-2.6878
73.2148
0.0000
-0.0000
-0.0000
0.0000
0.0000
0.0000
-30.6110
32.6878
15.6586

f1=

43.7836	-53.8986	-73.2148
43.7836	-53.8986	61.5316

f2=

-16.2164	26.1014	61.5316
-16.2164	26.1014	-3.7219

f3=

-20.5168	-22.8761	-53.7219
-44.5168	-4.8761	15.6586

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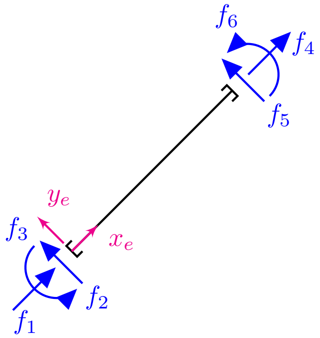
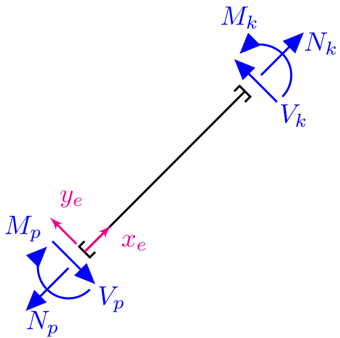
Force sign convention in FE

Classical FEM



CALFEM



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Script - *frame*.m - diagrams

```
% return to elements to compute nodal forces
f1=beam2s(Ex(1,:),Ey(1,:),ep,Qe(1,:),[0,0],7)
f2=beam2s(Ex(2,:),Ey(2,:),ep,Qe(2,:),[0,0],7)
f3=beam2s(Ex(3,:),Ey(3,:),ep,Qe(3,:),eq,21)
```

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Script - *frame*.m - diagrams

```
% return to elements to compute nodal forces
f1=beam2s(Ex(1,:),Ey(1,:),ep,Qe(1,:),[0,0],7)
f2=beam2s(Ex(2,:),Ey(2,:),ep,Qe(2,:),[0,0],7)
f3=beam2s(Ex(3,:),Ey(3,:),ep,Qe(3,:),eq,21)

% Deformed frame
figure(1)
eldraw2(Ex,Ey, [1,2,2]);
eldisp2(Ex,Ey,Qe,[1,4,1]);
axis([-1 7 -1 5]);
title('Displacements')
plotpar=[2 1];
```

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Script - *frame*.m - diagrams

```
% Normal forces
figure(2)
scal=scalfact2(Ex(3,:),Ey(3,:),f3(:,1),0.35);
eldia2(Ex(1,:),Ey(1,:),f1(:,1),plotpar,scal);
eldia2(Ex(2,:),Ey(2,:),f2(:,1),plotpar,scal);
eldia2(Ex(3,:),Ey(3,:),f3(:,1),plotpar,scal);
axis([-1 7 -1 5])
title('Normal forces')
```

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Script - *frame*.m - diagrams

```
% Normal forces
figure(2)
scal=scalfact2(Ex(3,:),Ey(3,:),f3(:,1),0.35);
eldia2(Ex(1,:),Ey(1,:),f1(:,1),plotpar,scal);
eldia2(Ex(2,:),Ey(2,:),f2(:,1),plotpar,scal);
eldia2(Ex(3,:),Ey(3,:),f3(:,1),plotpar,scal);
axis([-1 7 -1 5])
title('Normal forces')

% Shear forces
figure(3)
scal=scalfact2(Ex(1,:),Ey(1,:),f1(:,2),0.35);
eldia2(Ex(1,:),Ey(1,:),f1(:,2),plotpar,scal);
eldia2(Ex(2,:),Ey(2,:),f2(:,2),plotpar,scal);
eldia2(Ex(3,:),Ey(3,:),f3(:,2),plotpar,scal);
axis([-1 7 -1 5]);
title('Shear forces')
```

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Script - *frame*.m - diagrams

```
% Moments
figure(4)
scal=scalfact2(Ex(1,:),Ey(1,:),f1(:,3),0.35);
eldia2(Ex(1,:),Ey(1,:),f1(:,3),plotpar,scal);
eldia2(Ex(2,:),Ey(2,:),f2(:,3),plotpar,scal);
eldia2(Ex(3,:),Ey(3,:),f3(:,3),plotpar,scal);
axis([-1 7 -1 5]);
title('Bending moments');
```

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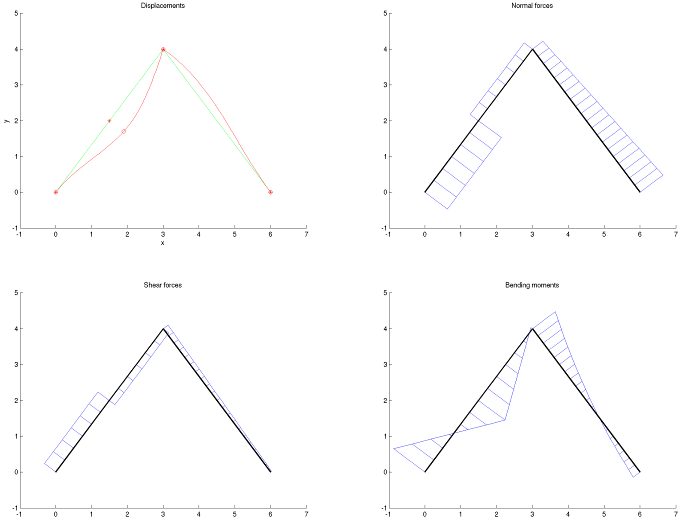
Diagrams



Project "The development of the didactic potential of Cracow University of Technology in the range of modern construction" is co-financed by the European Union
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