# GAUSSIAN MIXTURE MODEL FOR TIME SERIES-BASED STRUCTURAL DAMAGE DETECTION

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#### 1 INTRODUCTION

In recent years, various data-driven algorithms for structural damage detection have been developed using soft computing [2,5]. In most situations, damage detection can be solved as a novelty detection problem using unsupervised learning framework. In this work, the application of Gaussian mixture model (GMM) for solving damage detection problem is presented.

#### 2 DESCRIPTION OF DAMAGE DETECTION ALGORITHM

Damage detection is based on the premise that structural damage causes changes in measured vibration signals [4]. In case of time-series based damage detection, the acceleration signals from sensors can be modeled using time series models and the coefficients can be used as the damage-sensitive features.

In this paper, the autoregressive (AR) model of order p is used. The acceleration signal  $x_{\text{acc},i}(t)$  from sensor  $i^{\text{th}}$  is modeled by

$$x_{\text{acc},i}(t) = \sum_{k=1}^{p} \alpha_{ik} x_{\text{acc},i}(t-k) + \epsilon(t), \tag{1}$$

where  $\alpha_{ik}$  is  $k^{\text{th}}$  AR coefficient and  $\epsilon(t)$  is the residual term. The coefficients computed for the undamaged structure form then a statistical model of normality. After training, this model is subsequently applied for damage detection. In this work, only the first coefficient of the AR model for each sensor is used.

In this work, Gaussian mixture model (GMM) is applied because the feature vectors form two distinct clusters. GMM is defined as a superposition of K Gaussian densities and has the following form [1]:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x} | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k).$$
 (2)

<sup>&</sup>lt;sup>1</sup>Author would like to acknowledge support from the Polish Ministry of Science and Higher Education Grant "Applications of Bayesian machine learning methods in identification problems of experimental mechanics of materials and structures", No. N N506 250938.

One way to set the values of the Gaussian mixture distribution is to use maximum likelihood approach, maximizing the log of the likelihood function. It can be done with iterative optimization techniques like conjugate gradient method or alternatively using a powerful framework called expectation maximization (EM) [1].

#### 3 EXPERIMENTS AND RESULTS

The presented algorithm is validated using data generated from the FEM model of the ASCE benchmark steel frame [3]. The simulated structure's response to ambient vibrations in presence of damage is measured using 16 virtual accelerometers. The structural damage of the benchmark structure is simulated mainly by removing braces (loss of stiffness). There are six damage cases defined a priori. The data generation scripts in MATLAB are available on the web at http://mase.wustl.edu/asce.shm.

In numerical experiments for the undamaged structure, 580 patterns (16-dimensional feature vectors) were generated and modeled using GMM trained with EM algorithm. Then novelty threshold was estimated applying 10-fold cross-validation taking into account also patterns computed for damage scenarios (two severe and four minor). Finally, the presented algorithm was checked using testing patterns for all cases. Structural damage detection results are presented in Table 1.

Table 1: Results of testing the proposed algorithm for damage detection for various number of input variables (16 and 3 (after applying PCA)). In the last three columns the coefficients of success in % are presented for detecting three damage scenarios.

Number of inputs	undamaged[%]	severe damage[%]	minor damage[%]
16	60	98	67
3	53	98	50

### 4 FINAL REMARK

The proposed algorithm is able to detect severe damage even for only three input variables derived from PCA. But in the case of undamaged structure, it is possible at the expense of rather large number of false alarms (false-positive damage indications).

## REFERENCES

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