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LAMB WAVES AND PARTICLE FILTER FOR SEQUENTIAL ELASTIC CONSTANTS IDENTIFICATION OF ALUMINUM PLATE

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1 INTRODUCTION

In this work, the problem of sequential identification of elastic constants of a homogeneous isotropic aluminum plate is considered. The identification is based on the comparison of few parameters of two anti-symmetric dispersion curves A_0. The first one is computed from the Rayleigh-Lamb equation and the corresponding experimental curve is derived from the 2D-FFT of the velocity signals, measured at several points by a laser vibrometer [1].

The proposed sequential approach uses a particle filter algorithm, well described in the textbook by Russell and Norvig [2]. The particle filter has been recently applied in other identification problems. For example, Ching et al. has compared the particle filter and the extended Kalman filter in the problem of Bayesian state and parameter estimation of uncertain dynamical systems [3]. Nasrellah in [4], has proposed a strategy for combining finite element method and particle filter to tackle the problem of structural system parameter identification.

2 PARTICLE FILTER FOR STATE AND PARAMETER ESTIMATION

Particle filter is usually introduced in the probabilistic context for inference in dynamic Bayesian network (DBN) [2]. Dynamic Bayesian network is a Bayesian network which represents a temporal probability model. The well-known Kalman filter used to model linear discrete dynamic systems, is an example of dynamic Bayesian network with continuous variables and linear Gaussian conditional distributions. On the other hand, DBN can model arbitrary distribution in which the joint distribution over the sequence of K observed variables $\mathbf{y}_k$ and state (hidden) variables $\mathbf{x}_k$, $k = 1, ..., K$, is given by

$$p(\mathbf{x}_1, \ldots, \mathbf{x}_K, \mathbf{y}_1, \ldots, \mathbf{y}_K) = p(\mathbf{x}_1) \prod_{k=2}^{K} p(\mathbf{x}_k|\mathbf{x}_{k-1}) \prod_{k=1}^{K} p(\mathbf{y}_k|\mathbf{x}_k), \qquad (1)$$

where $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is the transition model and $p(\mathbf{y}_k|\mathbf{x}_k)$ is the observation model.

In the sequential estimation problems, we are mainly interested in recursive computations of the posterior distribution $p(\mathbf{x}_k|\mathbf{y}_k)$ but in general, exact inference is intractable so different approximate methods have been developed. The most successful algorithm for approximate inference is based on a sequential Monte Carlo sampling and approximating the posterior using N particles to obtain the empirical distribution $P_N(\mathbf{x}_k)$.

The basic PF algorithm starts with a population of N initial-state samples, created by sampling from the prior $p(\mathbf{x}_1)$. Then the update cycle is repeated for each time step [2]:

1. Each sample is propagated forward by sampling the next state value $\mathbf{x}_k$, given the current value $\mathbf{x}_{k-1}$ for the sample, based on the transition model $p(\mathbf{x}_k|\mathbf{x}_{k-1})$.

2. Each sample is weighted by the likelihood it assigns to the new evidence, $p(\mathbf{y}_k|\mathbf{x}_k)$.

3. The population is resampled to generate a new population of N samples. Each new sample is selected from the current population; the probability that a particular sample is selected is proportional to its weight. The new samples are unweighted.

3 RESULTS OF ELASTIC CONSTANTS SEQUENTIAL IDENTIFICATION

The presented identification problem was solved, by using the following assumptions. The elastic constants were treated as the time-independent state variables and the process model was only used to add a Gaussian noise. The independent prior distributions were assumed to be Gaussian. The observation model was based on the comparison of dispersion curves. The number of samples was N=1e3 and the number of steps for the particle filter was 1e2. True values for Young's modulus and Poisson ratio were $E = 69$GPa and $\mu = 0.33$, respectively.

The results of sequential identification of elastic constants are given in Fig. 1. On the left-hand side, the histogram for Young modulus is presented and on the right-hand side the corresponding histogram for Poisson ratio is shown. The histograms show that the proposed approach is able to identify the elastic constants with satisfactory accuracy. Moreover, it can be seen that this approach is naturally suited for uncertainty quantification.

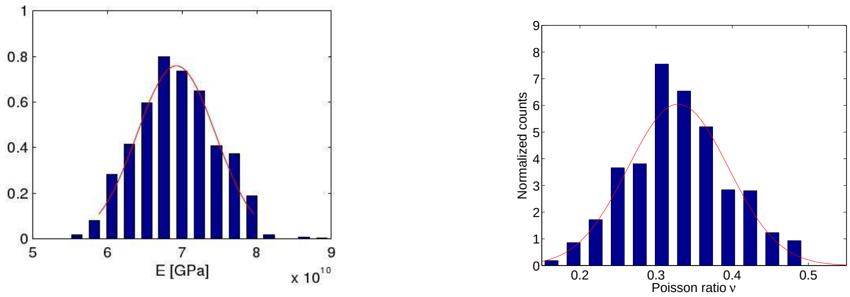

Figure 1: Posterior distribution for Young modulus (left) and Poisson ratio (right)

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