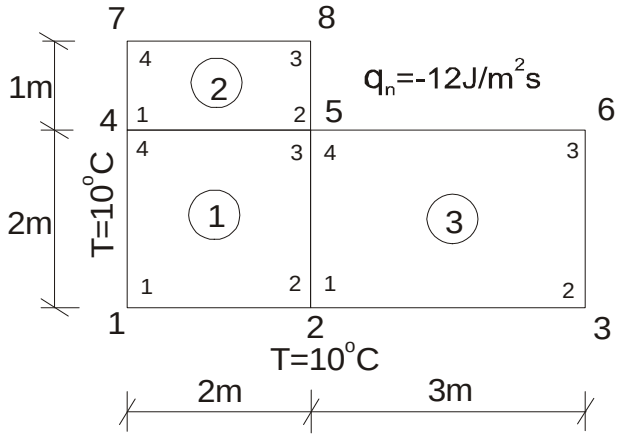
ORIGIN := 1




Funkcje kształtu

$$N_1(x,y,a,b) := \frac{(x-a)\cdot(y-b)}{a\cdot b}$$

$$N_2(x,y,a,b) := -\frac{x\cdot(y-b)}{a\cdot b}$$

$$N_3(x,y,a,b) := \frac{x\cdot y}{a\cdot b}$$

$$N_4(x,y,a,b) := -\frac{(x-a)\cdot y}{a\cdot b}$$

$k_x := 4$

$k_y := 7$

$$D := \begin{pmatrix} k_x & 0 \\ 0 & k_y \end{pmatrix}$$

gdzie a,b - wymiary elementów

Macierz pochodnych funkcji kształtu

$$B(x,y,a,b) := \begin{pmatrix} \frac{d}{dx}N_1(x,y,a,b) & \frac{d}{dx}N_2(x,y,a,b) & \frac{d}{dx}N_3(x,y,a,b) & \frac{d}{dx}N_4(x,y,a,b) \\ \frac{d}{dy}N_1(x,y,a,b) & \frac{d}{dy}N_2(x,y,a,b) & \frac{d}{dy}N_3(x,y,a,b) & \frac{d}{dy}N_4(x,y,a,b) \end{pmatrix}$$

Macierze przewodnictwa dla elementów

Element 1

$ii := 1..4$ $jj := 1..4$

$a_1 := 2$ $b_1 := 2$ $B_1(x,y) := B(x,y,a_1,b_1)$ $k_1(x,y) := B_1(x,y)^T\cdot D\cdot B_1(x,y)$

$$K_{1_{ii,jj}} := \int_0^{a_1}\int_0^{b_1} k_1(x,y)_{ii,jj}\,dy\,dx \qquad K_1 = \begin{pmatrix} 3.667 & -0.167 & -1.833 & -1.667 \\ -0.167 & 3.667 & -1.667 & -1.833 \\ -1.833 & -1.667 & 3.667 & -0.167 \\ -1.667 & -1.833 & -0.167 & 3.667 \end{pmatrix}$$

Element 2

$a_2 := 2$ $b_2 := 1$ $B_2(x,y) := B(x,y,a_2,b_2)$ $k_2(x,y) := B_2(x,y)^T\cdot D\cdot B_2(x,y)$

$$K_{2_{ii,jj}} := \int_0^{a_2}\int_0^{b_2} k_2(x,y)_{ii,jj}\,dy\,dx \qquad K_2 = \begin{pmatrix} 5.333 & 1.667 & -2.667 & -4.333 \\ 1.667 & 5.333 & -4.333 & -2.667 \\ -2.667 & -4.333 & 5.333 & 1.667 \\ -4.333 & -2.667 & 1.667 & 5.333 \end{pmatrix}$$

Element 3

$a_3 := 3$ $b_3 := 2$ $B_3(x,y) := B(x,y,a_3,b_3)$ $k_3(x,y) := B_3(x,y)^T\cdot D\cdot B_3(x,y)$

$$K_{3_{ii,jj}} := \int_0^{a_3}\int_0^{b_3} k_3(x,y)_{ii,jj}\,dy\,dx \qquad K_3 = \begin{pmatrix} 4.389 & 0.861 & -2.194 & -3.056 \\ 0.861 & 4.389 & -3.056 & -2.194 \\ -2.194 & -3.056 & 4.389 & 0.861 \\ -3.056 & -2.194 & 0.861 & 4.389 \end{pmatrix}$$

Macierze Boole'a

$$Bo_1 := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad Bo_2 := \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad Bo_3 := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Agregacja macierzy przewodnictwa

$$K := Bo_1^{T} \cdot K_1 \cdot Bo_1 + Bo_2^{T} \cdot K_2 \cdot Bo_2 + Bo_3^{T} \cdot K_3 \cdot Bo_3$$

wektor znanych wartości węzłowych temperatury

$$K = \begin{pmatrix} 3.667 & -0.167 & 0 & -1.667 & -1.833 & 0 & 0 & 0 \\ -0.167 & 8.056 & 0.861 & -1.833 & -4.722 & -2.194 & 0 & 0 \\ 0 & 0.861 & 4.389 & 0 & -2.194 & -3.056 & 0 & 0 \\ -1.667 & -1.833 & 0 & 9 & 1.5 & 0 & -4.333 & -2.667 \\ -1.833 & -4.722 & -2.194 & 1.5 & 13.389 & 0.861 & -2.667 & -4.333 \\ 0 & -2.194 & -3.056 & 0 & 0.861 & 4.389 & 0 & 0 \\ 0 & 0 & 0 & -4.333 & -2.667 & 0 & 5.333 & 1.667 \\ 0 & 0 & 0 & -2.667 & -4.333 & 0 & 1.667 & 5.333 \end{pmatrix} \quad Tz := \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 0 \\ 0 \\ 10 \\ 0 \end{pmatrix}$$

Wektor (obciążenia) strumienia ciepła

warunki brzegowe (1- znana wartość temperatury w węźle)

$$P_2 := \begin{pmatrix} 0 \\ \frac{-12 \cdot 1}{2} \\ \frac{-12 \cdot 1}{2} \\ 0 \end{pmatrix} \quad P_3 := \begin{pmatrix} 0 \\ 0 \\ \frac{-12 \cdot 3}{2} \\ \frac{-12 \cdot 3}{2} \end{pmatrix} \quad P := Bo_2^{T} \cdot P_2 + Bo_3^{T} \cdot P_3 \quad P = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -24 \\ -18 \\ 0 \\ -6 \end{pmatrix} \quad war := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Uwzględnienie warunków brzegowych

$$Pz := K \cdot Tz \qquad S := P - Pz \qquad i := 1..8 \qquad I := identity(8) \qquad Id_{i,i} := war_i \qquad Ip := I - Id$$

$$KK := Ip \cdot K \cdot Ip + Id$$

$$SS := Ip \cdot S$$

Wyliczenienie niewiadomych pierwotnych - temperatury w węzłach

$$T := KK^{-1} \cdot SS + Tz \qquad T = \begin{pmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 7.387 \\ 6.411 \\ 10 \\ 6.752 \end{pmatrix}$$

Wyliczenienie niewiadomych wtórnych - strumienia ciepła w węzłach

$$R := K \cdot T - P \qquad R = \begin{pmatrix} 4.79 \\ 20.214 \\ 16.699 \\ 4.742 \\ 2.842 \times 10^{-14} \\ -3.553 \times 10^{-15} \\ 1.555 \\ -2.132 \times 10^{-14} \end{pmatrix}$$

Powrót do elementów - Wyliczenie funkcji strumienia ciepła.
Wydruki dla środka i węzłów poszczególnych elementów

Element 1 $T_1 := Bo_1 \cdot T$

$$q_{x1}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_1(x,y) \right)_{1,i} \cdot T_{1_i} \right] \qquad q_{y1}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_1(x,y) \right)_{2,i} \cdot T_{1_i} \right]$$

$$q_{x1}\left( \frac{a_1}{2}, \frac{b_1}{2} \right) = 2.613 \qquad q_{y1}\left( \frac{a_1}{2}, \frac{b_1}{2} \right) = 4.573$$

$q_{x1}(0, b_1) = 5.226$ $q_{x1}(a_1, b_1) = 5.226$ $q_{y1}(0, b_1) = 0$ $q_{y1}(a_1, b_1) = 9.145$

$q_{x1}(0,0) = -3.553 \times 10^{-14}$ $q_{x1}(a_1, 0) = 0$ $q_{y1}(0,0) = -6.395 \times 10^{-14}$ $q_{y1}(a_1, 0) = 9.145$

Element 2 $T_2 := Bo_2 \cdot T$

$$q_{x2}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_2(x,y) \right)_{1,i} \cdot T_{2_i} \right] \qquad q_{y2}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_2(x,y) \right)_{2,i} \cdot T_{2_i} \right]$$

$$q_{x2}\left( \frac{a_2}{2}, \frac{b_2}{2} \right) = 5.861 \qquad q_{y2}\left( \frac{a_2}{2}, \frac{b_2}{2} \right) = 2.223$$

$q_{x2}(0, b_2) = 6.496$ $q_{x2}(a_2, b_2) = 6.496$ $q_{y2}(0, b_2) = 0$ $q_{y2}(a_2, b_2) = 4.445$

$q_{x2}(0,0) = 5.226$ $q_{x2}(a_2, 0) = 5.226$ $q_{y2}(0,0) = 6.395 \times 10^{-13}$ $q_{y2}(a_2, 0) = 4.445$

Element 3 $T_3 := Bo_3 \cdot T$

$$q_{x3}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_3(x,y) \right)_{1,i} \cdot T_{3_i} \right] \qquad q_{y3}(x,y) := \sum_{i=1}^{4} \left[ \left( -D \cdot B_3(x,y) \right)_{2,i} \cdot T_{3_i} \right]$$

$$q_{x3}\left( \frac{a_3}{2}, \frac{b_3}{2} \right) = 0.65 \qquad q_{y3}\left( \frac{a_3}{2}, \frac{b_3}{2} \right) = 10.853$$

$q_{x3}(0, b_3) = 1.301$ $q_{x3}(a_3, b_3) = 1.301$ $q_{y3}(0, b_3) = 9.145$ $q_{y3}(a_3, b_3) = 12.56$

$q_{x3}(0,0) = 6.164 \times 10^{-13}$ $q_{x3}(a_3, 0) = 3.553 \times 10^{-15}$ $q_{y3}(0,0) = 9.145$ $q_{y3}(a_3, 0) = 12.56$

Rysowanie mapy rozkładu temperatury dla całego układu

$$Ng_1(x,y) := N_1(x,y,a_1,b_1)\cdot T_{1_1} + N_2(x,y,a_1,b_1)\cdot T_{1_2} + N_3(x,y,a_1,b_1)\cdot T_{1_3} + N_4(x,y,a_1,b_1)\cdot T_{1_4}$$

$$Ng_{2p}(x,y) := N_1(x,y-b_1,a_2,b_2)\cdot T_{2_1} + N_2(x,y-b_1,a_2,b_2)\cdot T_{2_2}$$

$$Ng_2(x,y) := Ng_{2p}(x,y) + N_3(x,y-b_1,a_2,b_2)\cdot T_{2_3} + N_4(x,y-b_1,a_2,b_2)\cdot T_{2_4}$$

$$Ng_{3p}(x,y) := N_1(x-a_1,y,a_3,b_3)\cdot T_{3_1} + N_2(x-a_1,y,a_3,b_3)\cdot T_{3_2}$$

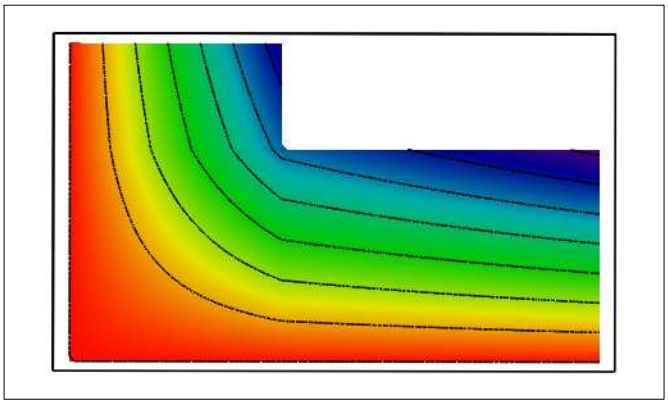
$$Ng_3(x,y) := Ng_{3p}(x,y) + N_3(x-a_1,y,a_3,b_3)\cdot T_{3_3} + N_4(x-a_1,y,a_3,b_3)\cdot T_{3_4}$$

$$Ng(x,y) := \text{if}\left(x \le a_1, \text{if}\left(y \le b_1, Ng_1(x,y), Ng_2(x,y)\right), \text{if}\left(y \le b_1, Ng_3(x,y), 0\right)\right)$$

$i := 1..101$ $\quad j := 1..61$ $\quad xx_i := (i-1)\cdot 0.05$ $\quad yy_j := (j-1)\cdot 0.05$

$$XX_{i,j} := \text{if}\left[\left(yy_j > b_1\right)\cdot\left(xx_i > a_1\right), 0, xx_i\right] \qquad YY_{i,j} := \text{if}\left[\left(yy_j > b_1\right)\cdot\left(xx_i > a_1\right), 0, yy_j\right]$$

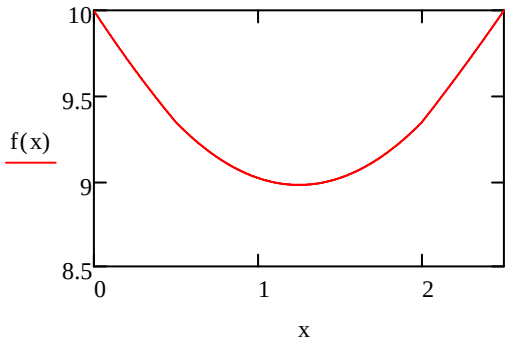
$$T_{i,j} := Ng\left(XX_{i,j}, YY_{i,j}\right)$$



$(XX, YY, T)$

Wykres dla przekroju y=2.5-x

$$f(x) := Ng(x, 2.5-x)$$




Wykres dla przekroju y=1.5

$$f_2(x) := Ng(x, 1.5)$$