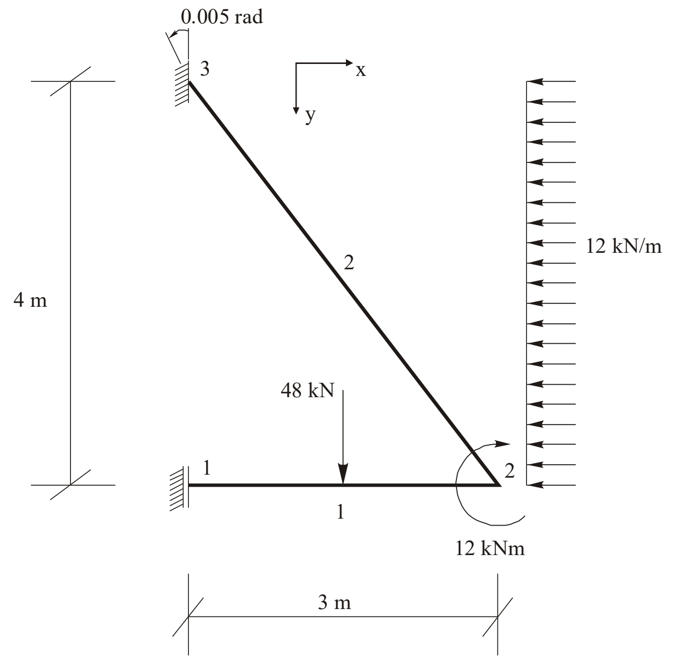
Wzory na zastępniki, macierz sztywności i macierz transformacji



$$ORIGIN := 1$$

$$z_s(p,l) := \begin{pmatrix} 0 \\ \frac{p}{2} \\ \frac{p \cdot l}{8} \\ 0 \\ \frac{p}{2} \\ \frac{-p \cdot l}{8} \end{pmatrix} \qquad z_c(p,l) := \begin{pmatrix} 0 \\ \frac{p \cdot l}{2} \\ \frac{p \cdot l^2}{12} \\ 0 \\ \frac{p \cdot l}{2} \\ \frac{-p \cdot l^2}{12} \end{pmatrix}$$

$$T(\cos,\sin) := \begin{pmatrix} \cos & \sin & 0 & 0 & 0 & 0 \\ -\sin & \cos & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos & \sin & 0 \\ 0 & 0 & 0 & -\sin & \cos & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Kl(EA,EI,l) := \begin{pmatrix} \frac{EA}{l} & 0 & 0 & \frac{-EA}{l} & 0 & 0 \\ 0 & \frac{12 \cdot EI}{l^3} & \frac{6 \cdot EI}{l^2} & 0 & \frac{-12 \cdot EI}{l^3} & \frac{6 \cdot EI}{l^2} \\ 0 & \frac{6 \cdot EI}{l^2} & \frac{4 \cdot EI}{l} & 0 & -\frac{6 \cdot EI}{l^2} & \frac{2 \cdot EI}{l} \\ \frac{-EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & \frac{-12 \cdot EI}{l^3} & -\frac{6 \cdot EI}{l^2} & 0 & \frac{12 \cdot EI}{l^3} & -\frac{6 \cdot EI}{l^2} \\ 0 & \frac{6 \cdot EI}{l^2} & \frac{2 \cdot EI}{l} & 0 & -\frac{6 \cdot EI}{l^2} & \frac{4 \cdot EI}{l} \end{pmatrix}$$

Dane wejściowe

$$EA := 1.5 \cdot 10^5 \qquad EI := 3375 \qquad top := \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$x_1 := 3 \quad y_1 := 0 \quad l_1 := \sqrt{x_1^{\ 2} + y_1^{\ 2}} \quad cos1 := \frac{x_1}{l_1} \quad sin1 := \frac{y_1}{l_1}$$

$$x_2 := -3 \quad y_2 := -4 \quad l_2 := \sqrt{x_2^{\ 2} + y_2^{\ 2}} \quad cos2 := \frac{x_2}{l_2} \quad sin2 := \frac{y_2}{l_2}$$

Macierze Boole'a

$$B(el) := \left| \begin{array}{l} B_{6,9} \leftarrow 0 \\ \text{for } i \in 1..3 \\ \quad \left| \begin{array}{l} B_{i,3 \cdot (top_{el,1}-1)+i} \leftarrow 1 \\ B_{i+3,3 \cdot (top_{el,2}-1)+i} \leftarrow 1 \end{array} \right. \end{array} \right.$$

$$B(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad B(2) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Macierze sztywności w konfiguracji lokalnej

$$Kl_1 := Kl(EA, EI, l_1) \qquad Kl_1 = \begin{pmatrix} 5\times10^4 & 0 & 0 & -5\times10^4 & 0 & 0 \\ 0 & 1.5\times10^3 & 2.25\times10^3 & 0 & -1.5\times10^3 & 2.25\times10^3 \\ 0 & 2.25\times10^3 & 4.5\times10^3 & 0 & -2.25\times10^3 & 2.25\times10^3 \\ -5\times10^4 & 0 & 0 & 5\times10^4 & 0 & 0 \\ 0 & -1.5\times10^3 & -2.25\times10^3 & 0 & 1.5\times10^3 & -2.25\times10^3 \\ 0 & 2.25\times10^3 & 2.25\times10^3 & 0 & -2.25\times10^3 & 4.5\times10^3 \end{pmatrix}$$

$$Kl_2 := Kl(EA, EI, l_2) \qquad Kl_2 = \begin{pmatrix} 3\times10^4 & 0 & 0 & -3\times10^4 & 0 & 0 \\ 0 & 324 & 810 & 0 & -324 & 810 \\ 0 & 810 & 2.7\times10^3 & 0 & -810 & 1.35\times10^3 \\ -3\times10^4 & 0 & 0 & 3\times10^4 & 0 & 0 \\ 0 & -324 & -810 & 0 & 324 & -810 \\ 0 & 810 & 1.35\times10^3 & 0 & -810 & 2.7\times10^3 \end{pmatrix}$$

Macierze transformacji i sztywności w konfiguracji globalnej dla elementów

$$T_1 := T(\cos1, \sin1) \qquad T_2 := T(\cos2, \sin2)$$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad T_2 = \begin{pmatrix} -0.6 & -0.8 & 0 & 0 & 0 & 0 \\ 0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6 & -0.8 & 0 \\ 0 & 0 & 0 & 0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_1 := T_1^T \cdot Kl_1 \cdot T_1$$

$$K_1 = \begin{pmatrix} 5\times10^4 & 0 & 0 & -5\times10^4 & 0 & 0 \\ 0 & 1.5\times10^3 & 2.25\times10^3 & 0 & -1.5\times10^3 & 2.25\times10^3 \\ 0 & 2.25\times10^3 & 4.5\times10^3 & 0 & -2.25\times10^3 & 2.25\times10^3 \\ -5\times10^4 & 0 & 0 & 5\times10^4 & 0 & 0 \\ 0 & -1.5\times10^3 & -2.25\times10^3 & 0 & 1.5\times10^3 & -2.25\times10^3 \\ 0 & 2.25\times10^3 & 2.25\times10^3 & 0 & -2.25\times10^3 & 4.5\times10^3 \end{pmatrix}$$

$$K_2 := T_2^T \cdot Kl_2 \cdot T_2$$

$$K_2 = \begin{pmatrix} 1.101\times10^4 & 1.424\times10^4 & 648 & -1.101\times10^4 & -1.424\times10^4 & 648 \\ 1.424\times10^4 & 1.932\times10^4 & -486 & -1.424\times10^4 & -1.932\times10^4 & -486 \\ 648 & -486 & 2.7\times10^3 & -648 & 486 & 1.35\times10^3 \\ -1.101\times10^4 & -1.424\times10^4 & -648 & 1.101\times10^4 & 1.424\times10^4 & -648 \\ -1.424\times10^4 & -1.932\times10^4 & 486 & 1.424\times10^4 & 1.932\times10^4 & 486 \\ 648 & -486 & 1.35\times10^3 & -648 & 486 & 2.7\times10^3 \end{pmatrix}$$

Agregacja macierzy sztywności

$$K := B(1)^T \cdot K_1 \cdot B(1) + B(2)^T \cdot K_2 \cdot B(2)$$

$$K = \begin{pmatrix} 5\times10^4 & 0 & 0 & -5\times10^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5\times10^3 & 2.25\times10^3 & 0 & -1.5\times10^3 & 2.25\times10^3 & 0 & 0 & 0 \\ 0 & 2.25\times10^3 & 4.5\times10^3 & 0 & -2.25\times10^3 & 2.25\times10^3 & 0 & 0 & 0 \\ -5\times10^4 & 0 & 0 & 6.101\times10^4 & 1.424\times10^4 & 648 & -1.101\times10^4 & -1.424\times10^4 & 648 \\ 0 & -1.5\times10^3 & -2.25\times10^3 & 1.424\times10^4 & 2.082\times10^4 & -2.736\times10^3 & -1.424\times10^4 & -1.932\times10^4 & -486 \\ 0 & 2.25\times10^3 & 2.25\times10^3 & 648 & -2.736\times10^3 & 7.2\times10^3 & -648 & 486 & 1.35\times10^3 \\ 0 & 0 & 0 & -1.101\times10^4 & -1.424\times10^4 & -648 & 1.101\times10^4 & 1.424\times10^4 & -648 \\ 0 & 0 & 0 & -1.424\times10^4 & -1.932\times10^4 & 486 & 1.424\times10^4 & 1.932\times10^4 & 486 \\ 0 & 0 & 0 & 648 & -486 & 1.35\times10^3 & -648 & 486 & 2.7\times10^3 \end{pmatrix}$$

Wektor zastępników

$$zl_1 := z_s(48,3) \qquad zl_2 := z_c(-12,4)$$

$$z_1 := zl_1 \qquad z_2 := (T(0,-1))^T \cdot zl_2 \qquad z := B(1)^T \cdot z_1 + B(2)^T \cdot z_2$$

$$z_1 = \begin{pmatrix} 0 \\ 24 \\ 18 \\ 0 \\ 24 \\ -18 \end{pmatrix} \qquad zl_2 = \begin{pmatrix} 0 \\ -24 \\ -16 \\ 0 \\ -24 \\ 16 \end{pmatrix} \qquad z_2 = \begin{pmatrix} -24 \\ 0 \\ -16 \\ -24 \\ 0 \\ 16 \end{pmatrix} \qquad z = \begin{pmatrix} 0 \\ 24 \\ 18 \\ -24 \\ 24 \\ -34 \\ -24 \\ 0 \\ 16 \end{pmatrix}$$

Wektor sił węzłowych

$$p_9 := 0 \qquad p_6 := 12 \qquad p = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Wektor obciążenia kinematycznego

$$d_{wb_9} := 0 \qquad d_{wb_9} := -0.005 \qquad d_{wb} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5\times10^{-3} \end{pmatrix}$$

Wektor prawej strony równania MES

$s := z + p - K \cdot d_{wb}$

$$s = \begin{pmatrix} 0 \\ 24 \\ 18 \\ -20.76 \\ 21.57 \\ -15.25 \\ -27.24 \\ 2.43 \\ 29.5 \end{pmatrix}$$

Warunki brzegowe (1 - zablokowany stopień swobody)

$$war := \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Uwzględnienie warunków brzegowych

$i := 1..9 \quad I := identity(9) \quad Id_{i,i} := war_i \quad Ip := I - Id \quad KK := Ip \cdot K \cdot Ip + Id \quad ss := Ip \cdot s$

$$KK = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5 \times 10^3 & 0 & 0 & -1.5 \times 10^3 & 2.25 \times 10^3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.101 \times 10^4 & 1.424 \times 10^4 & 648 & 0 & 0 & 0 \\ 0 & -1.5 \times 10^3 & 0 & 1.424 \times 10^4 & 2.082 \times 10^4 & -2.736 \times 10^3 & 0 & 0 & 0 \\ 0 & 2.25 \times 10^3 & 0 & 648 & -2.736 \times 10^3 & 7.2 \times 10^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad ss = \begin{pmatrix} 0 \\ 24 \\ 0 \\ -20.76 \\ 21.57 \\ -15.25 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Rozwiązanie układu równań - wektor przemieszczeń węzłowych i wektor reakcji

$d := KK^{-1} \cdot ss + d_{wb} \qquad r := K \cdot d - z - p$

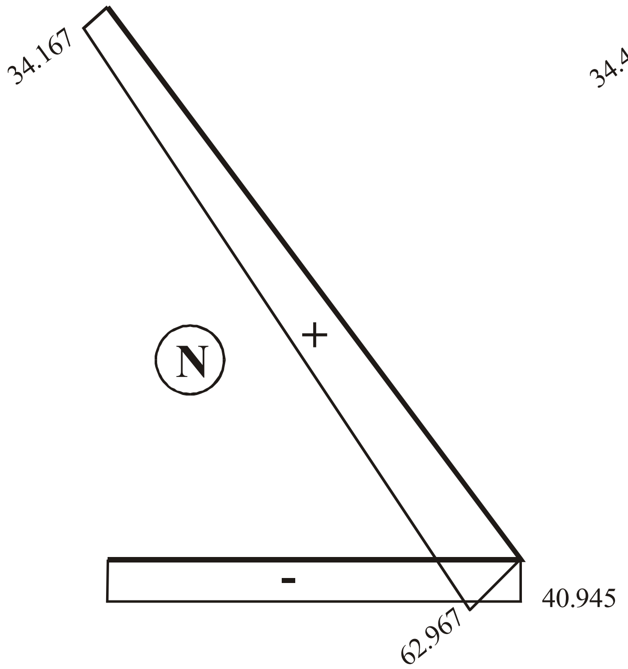
$$d = \begin{pmatrix} 0 \\ 0.038 \\ 0 \\ -8.189 \times 10^{-4} \\ 2.638 \times 10^{-3} \\ -0.013 \\ 0 \\ 0 \\ -5 \times 10^{-3} \end{pmatrix} \qquad r = \begin{pmatrix} 40.945 \\ -7.105 \times 10^{-15} \\ 32.54 \\ 7.105 \times 10^{-15} \\ 7.105 \times 10^{-15} \\ -1.421 \times 10^{-14} \\ 7.055 \\ -48 \\ -48.761 \end{pmatrix}$$

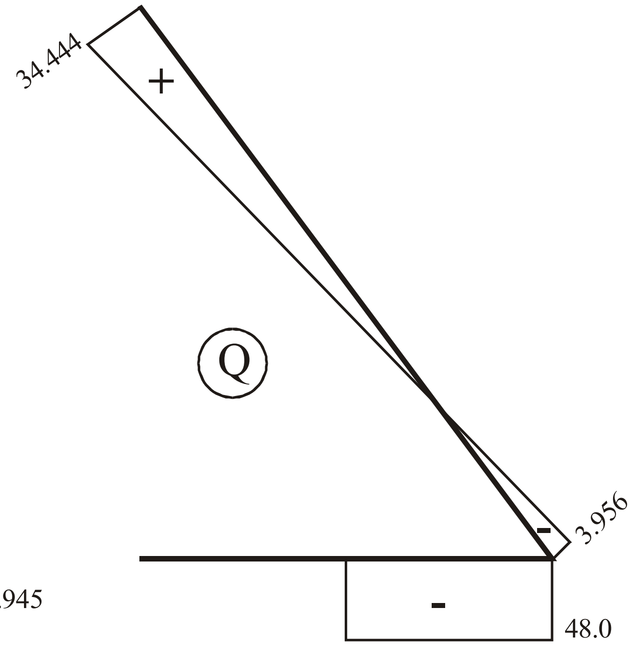
Powrót do elementów - obliczenie sił przywęzłowych w elementach

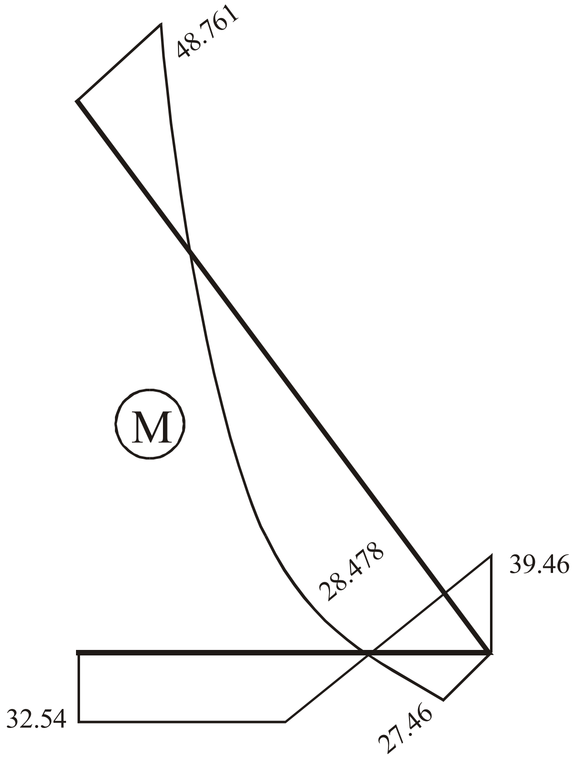
$fl_1 := T_1 \cdot (K_1 \cdot B(1) \cdot d - z_1) \qquad fl_2 := T_2 \cdot (K_2 \cdot B(2) \cdot d - z_2)$

$$fl_1 = \begin{pmatrix} 40.945 \\ -7.105 \times 10^{-15} \\ 32.54 \\ -40.945 \\ -48 \\ 39.46 \end{pmatrix} \qquad fl_2 = \begin{pmatrix} -62.967 \\ 3.956 \\ -27.46 \\ 34.167 \\ 34.444 \\ -48.761 \end{pmatrix}$$

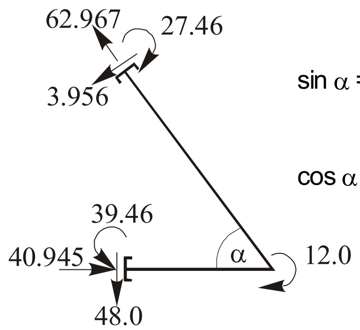
Wykresy sił przekrojowych







Równowaga węzła



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$\sum_{x=0}$ $$40.945 - 62.967\cdot\frac{3}{5} - 3.956\cdot\frac{4}{5} = 0$$

$\sum_{y=0}$ $$-48 + 62.967\cdot\frac{4}{5} - 3.956\cdot\frac{3}{5} = 3.553\times 10^{-15}$$

$\sum_{M=0}$ $$27.46 + 12 - 39.46 = 0$$