Geometric modelling

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Presentation outline

- Stages of FEM algorithm
- Geometric modelling

Stages of FEM algorithm

Stage I - preprocessing

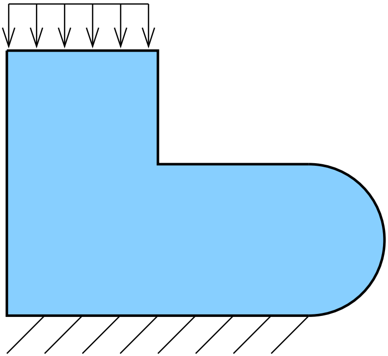
- Description of the geometry and assigning attributes like materials, loads, boundary conditions to the components of the geometric model.
- Discretisation of the model and its attributes – mesh generation.
- Building of matrices and vectors describing the elements.
- Visualisation of the geometric and discretisation data for control purposes.

Stage II - solution of a system of linear algebraic equations

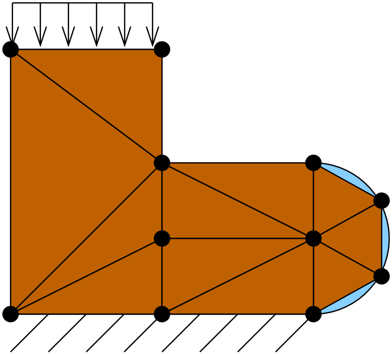
Stage III - postprocessing

Input data for preprocessing stage

geometric model



coarse mesh model



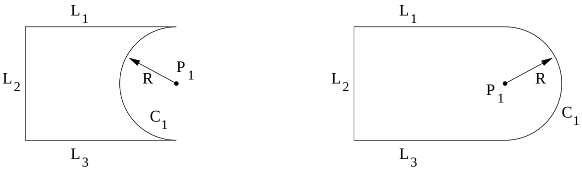
Description of a geometric object

Information necessary for defining geometric object can be divided into two categories

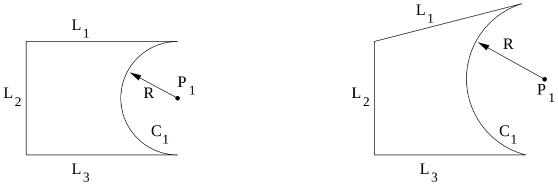
- geometric information – coordinates and dimensions of the object and its components,
- topological information – the relations between components the object consists of, for instance which vertices are connected by edges.

Geometry versus topology

The same geometry but different topology:



The same topology but different geometry:



Geometric representation properties

When looking at geometric object representation we should pay attention if it is: **complete**, **valid**, **unambiguous**.

- A representation is complete if it allows to classify a point as being inside, outside or on the object boundary.
- A representation **valid** if it is possible to build physical model on it, e.g. it does not contain free edges or faces.
- A representation is **valid**, if it can be interpreted only in one way.

Models of 3D objects

Exact

Topological and geometrical properties are expressed in mathematically exact way.

Examples:

- Wireframe models
- Surface models
- Solid models
  - CSG
  - B-Rep
  - Implicit functions models

Approximate

A model is represented by a set of simple primitives.

Examples:

- Surface meshes
- Voxel representation (voxel – 3D pixel)

Difficulties in 3D geometric modelling

Exact modelling

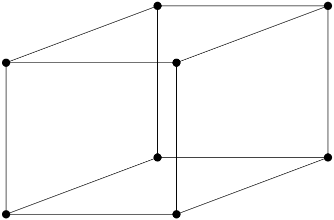
- complex data structures
- expensive algorithms
- many (often incompatible) data formats
- difficulties in data acquisition
- required complex data processing in order to visualise the model

Approximate modelling

- because of approximation some details are lost
- huge data structures in order to provide required resolution
- easy to introduce inconsistencies (e.g. cracks or overlapping elements)

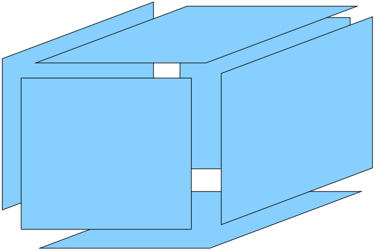
Wireframe modelling

- One of the first techniques introduced in the beginning of the 1960s.
- An object is represented as a set of edges joining the nodes lying on the object's surface.
- The simplest thus the fastest technique in terms of processing speed.



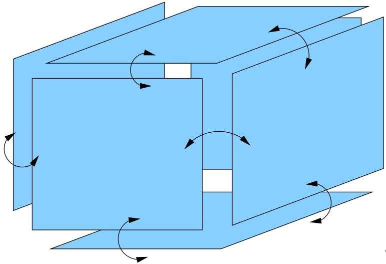
Surface modelling

- Developed in the end of the 1960s as the extension of wireframe modelling. It enriches the object description by attaching mathematical description of object's surfaces.
- It can be used for both finite volume models (closed surfaces) and for infinite volume models (open surfaces, e.g. infinite cylinders).



Solid modelling

- Introduced in the beginning of the 1970s.
- It is a complete description of an object as a rigid body.
- It distinguishes object interior space
- It enables analysis of the geometric properties like volumes, inertia moments, and so on.



The arrows mean stored information about relations between the surfaces.

Solid modelling – key points

- Using solid modelling one gets complete, valid and unambiguous description of physical objects.
- A singe object can be represented with different solid modelling techniques but interpretation of such representations gives exactly the same physical object.
- Solid modelling enables the analysis of properties related to the object volume.
- Processing of solid models is the most expensive in comparison to other modelling techniques but with today computers the cost of it is not prohibitive any more.

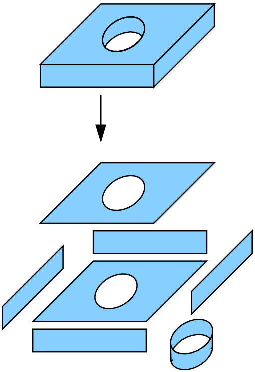
Different solid representations

The most common approaches to solid representation:

- Half-Spaces.
- Boundary representation (B-Rep).
- CSG (Constructive Solid Geometry).
- Sweeping representation (2.5D).
- Algebraic representations (e.g. isosurfaces)
- Spatial decomposition representations
  - Cell decomposition.
  - Voxel representation.
  - Binary Space Partitioning trees.

Boundary representation

In boundary representation the solid is described by a set of faces which are arranged as an oriented manifold surface. The surface is oriented if we can distinguish two sides of the surface. Each face can be flat or curved.



Elements of boundary representation

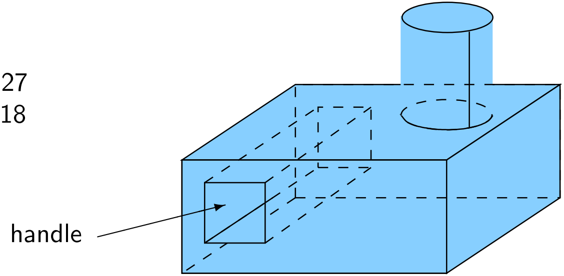
- Vertex V – point in 3D space
- Edge E – segment of a curve defined by two vertices
- Face F – manifold surface bounded by the edges
- Loop L – closed sequence of edges located in the face interior

Additionally one used the following concepts

- Void – a cavity completely enclosed in the solid and having no common points with solid boundary
- Handle – an open hole. Number of such holes is termed genus

Elements of boundary representation – examples

faces $F = 6 + 4 + 2 = 12$
edges $E = 12 + 12 + 3 = 27$
vertices $V = 8 + 8 + 2 = 18$
loops $L = 2 + 1 = 3$
handles $G = 1$
solids $B = 1$



Side note

By the Euler–Poincare theorem topologically valid solid should satisfy the following equation:

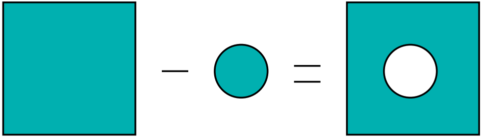
$$F - E + V - L = 2(B - G)$$

where V, E, F, L, B i G denote the respective number of solid elements.

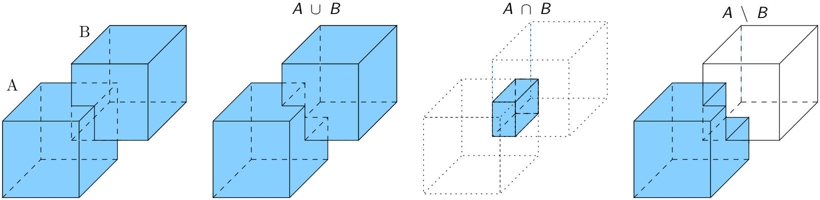
CSG

CSG = Constructive Solid Geometry

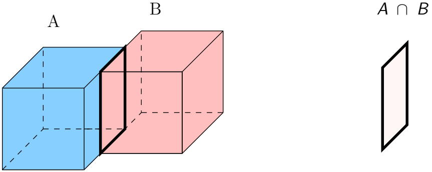
In CSG the solid is constructed by applying Boolean operations to a set of basic geometric primitives (cube, cylinder, sphere, cone, etc). The primitives as well as the results of Boolean operations can be modified by geometric transformations like, translation, rotation and scaling. In CSG modelling an object is stored as a binary tree with internal nodes representing Boolean operations and leaves representing the primitives used.



Boolean operations

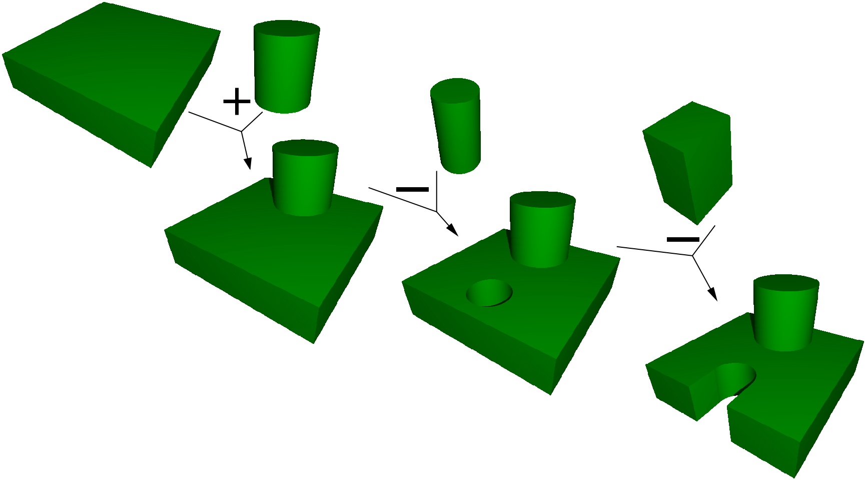


Problem with ordinary Boolean operations: the result of Boolean operations on two valid solids might not be a solid object. Example:



Solution: regularised Boolean operations.

CSG – example



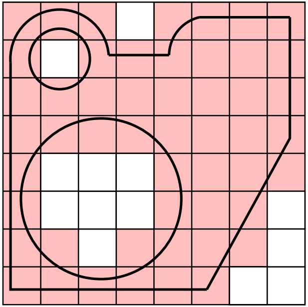
Spatial decomposition representations

Spatial decomposition representations are based on splitting a solid into a set of adjacent solids which are simpler than the original solid The shape of decomposition solid, their parametrisation and the resolution of the decomposition depend on the envisioned usage of the model. We can distinguish the following representations:

- Cell decomposition,
- Voxel representation,
- Binary Space Partitioning trees, including:
  - Kd-tree,
  - octree.

Voxel representation

In voxel representation a solid is decomposed into a set of identical cells arranged as a regular structured grid. The cells are called voxels. The most common voxel is cube. In voxel representation only cell that are located in the solid interior are saved, thus the solid can be only approximated by a voxel model



Subdivision trees

One of the shortcoming of voxel representation is the huge memory consumption related to the use of constant regular grid. A remedy to this is the use of hierarchical data structures based on subdivision trees:

- BSP trees – resulting from recursive bisection of a k-dimensional space with k-1 dimensional hyperplane
- Kd trees – special case of BSP trees with subdivision planes aligned with coordinate planes
- octree – resulting from recursive subdivision of the space into eigth (8) identical cubes. Can be regarded as a special case of Kd trees.

Quadtree

Quadtree is a 2-dimensional version of octree data structure.